Name:

Math 116 Exam 3

Directions: WRITE YOUR NAME ON THIS EXAM! Except where indicated, merely finding the answer to a problem is not enough to receive full credit; you must show how you arrived at that answer. Decimal approximations up to four decimal points are acceptable unless otherwise indicated.

1) a) (3 points) Parameterize a generic function $y = f(x)$.

b) (2 points) Is there more than one polar representation for a given point (x, y) in Cartesian coordinates? If so, how many different representations are there?

c) (4 points) Given a parametric function $f(t) = \langle x(t), y(t) \rangle$ that is differentiable at $t = 2$, what does the quantity $y'(2)/x'(2)$ represent if $x'(2) \neq 0$? What can you say if $x'(2) = 0$?

2) a) (6 points) What are the rectangular (Cartesian) coordinates of the polar point $\left(-3, \frac{7\pi}{4}\right)$?

b) (10 points) Give two representations in polar coordinates for the rectangular point $(-3, -3\sqrt{3})$

3) a) (22 points) Determine a parametric equation for the tangent line to the parametric curve given by $f(t) = \langle (2t)^t, \arctan(e^{t-1}) \rangle$ at the point $(2, \pi/4)$.

b) (2 points) What is the value of the slope of the tangent line from a)?

4) Recall that the area inside a polar curve $r = f(\theta)$ from $\theta = \theta_0$ to $\theta = \theta_1$ is given by

$$A = \frac{1}{2} \int_{\theta_0}^{\theta_1} (f(\theta))^2 \, d\theta.$$

a) (12 points) Use this formula to compute the area inside $r = \sin(\theta)$ from $\theta = 0$ to $\theta = \pi$.

b) (13 points) Find an equation in Cartesian coordinates for $r = \sin(\theta)$. Use this to confirm that the area from part a) is $\pi/4$.

5) Let $f(t) = \langle t^2 \sin(t), t^2 \cos(t) \rangle$.

a) (14 points) Set up an equation for the arclength of the graph of f from $t = 0$ to $t = \sqrt{5}$.

b) (12 points) Find the arclength of the portion of the curve described in part a).