Name:

Math 116 Exam 3

Directions: WRITE YOUR NAME ON THIS EXAM! Except where indicated, merely finding the answer to a problem is not enough to receive full credit; you must show how you arrived at that answer. Decimal approximations up to four decimal points are acceptable unless otherwise indicated.

1) Suppose you have a power series whose center is $c = 9$ and whose radius of convergence is $R = 2$.

a) (4 points) Find four numbers for which the power series definitely converges.

b) (3 points) Find three numbers for which the power series definitely diverges.

c) (2 points) Find the only two numbers for which you can't tell whether the series converges or diverges.

2) Consider the power series $\sum_{n=1}^{\infty} \frac{(2x-18)^n}{4^n \arctan(n)}$.

a) (2 points) What is the center of this series?

b) (15 points) Find the radius of convergence of this series.

3) (13 points) Determine the limit of the sequence

$$\left(\left(1-\frac{24}{n}\right)^{3n}\right)_{n=1}^{\infty}.$$

4) a) (3 points) Provide all numbers for which $\displaystyle\sum_{n=0}^{\infty} x^n = \frac{1}{1-x}$.

b) (9 points) Find the sum of $\displaystyle\sum_{n=1}^{\infty} \frac{(-8)^n}{3^{4n}}$ or show it diverges.

c) (11 points) Find the sum of $\displaystyle\sum_{n=0}^{\infty} \frac{\pi^{2n+1}}{n!}$ or show it diverges.

5) a) (6 points) Define what it means for $\sum_{n=1}^{\infty} a_n$ to converge to a real number.

b) (14 points) Using the definition of convergence for a series, find the sum of $\sum_{n=2}^{\infty} \left(\frac{1}{\ln(n)} - \frac{1}{\ln(n+1)} \right)$ or show the series diverges.

6) (18 points) Let $f(x) = \sum_{n=1}^{\infty} \frac{e^n \left(\frac{1}{e} - x\right)^n}{n}$ for $0 < x < 2/e$. Show that f satisfies the differential equation

$$y' + xy'' = 0.$$