W' 23 Exam 3

1) a) $0, 1, 3/4$

b) $2, 4, -10$

c) $3/2, -1/2$

2) a) $c = 6$

b) $$a_n = \frac{(3x-18)^n}{\sqrt{2n+1}}$$

$$a_{n+1} = \frac{(3x-18)^{n+1}}{\sqrt{2(n+1)+1}} = \frac{(3x-18)^{n+1}}{\sqrt{2n+3}}$$

$$\frac{a_{n+1}}{a_n} = \frac{(3x-18)^{n+1}}{\sqrt{2n+3}} \cdot \frac{\sqrt{2n+1}}{(3x-18)^n}$$

$$= \frac{(3x-18)^{n+1}}{(3x-18)^n} \cdot \frac{\sqrt{2n+1}}{\sqrt{2n+3}}$$

$$= (3x-18)\sqrt{\frac{2n+1}{2n+3}}$$

$$\left|\frac{a_{n+1}}{a_n}\right| = |3x-18| \sqrt{\frac{2n+1}{2n+3}}$$

$$\lim_{n\to\infty} \left|\frac{a_{n+1}}{a_n}\right| = \lim_{n\to\infty} |3x-18| \cdot \sqrt{\frac{2n+1}{2n+3}}$$

$$= |3x-18| \sqrt{\lim_{n\to\infty} \frac{2n+1}{2n+3}}$$

$$\lim_{n\to\infty} \frac{2n+1}{2n+3} \overset{\text{l'H}}{=} \lim_{n\to\infty} \frac{2}{2} = 1,$$

we get

$$|3x-18| < 1$$

$$-1 < 3x-18 < 1$$

$$-1/3 < x-6 < 1/3$$

$$R = 2/3/2 = 1/3$$

3) a) $$\frac{7^{n-4}}{2^{3n-7}} = \frac{2^7}{7^4} \cdot \frac{7^n}{2^{3n}} =$$

$$\frac{2^7}{7^4} \cdot \frac{7^n}{8^n} = a_n$$

$$a_{n+1} = \frac{7^{n+1}}{8^{n+1}} \cdot \frac{2^7}{7^4}$$

$$\frac{a_{n+1}}{a_n} = \frac{7^{n+1}}{8^{n+1}} \cdot \cancel{\frac{2^7}{7^4}} \cdot \cancel{\frac{7^4}{2^7}} \cdot \frac{8^n}{7^n}$$

$$= \frac{7^{n+1}}{7^n} \cdot \frac{8^n}{8^{n+1}}$$

$$= \frac{7}{8}$$, geometric

$\left|\frac{7}{8}\right| = 7/8 < 1$, so

the series converges to

$$\frac{\frac{7^{3-4}}{2^{9-7}}}{1-\frac{7}{8}} = \frac{\frac{7^{-1}}{4}}{(1/8)}$$

$$= 2/7$$

b) $a_n = \dfrac{1}{10^{n^2}}$

$$a_{n+1} = \frac{1}{10^{(n+1)^2}} = \frac{1}{10^{n^2+2n+1}}$$

$$\frac{a_{n+1}}{a_n} = \frac{10^{n^2}}{10^{n^2+2n+1}} = 10^{-2n-1}$$

not geometric

4)

$$\left(\frac{n}{n+7}\right)^{6n}$$

$$= \left(\frac{n+7-7}{n+7}\right)^{6n}$$

$$= \left(1-\frac{7}{n+7}\right)^{6n}$$

$$= e^{\ln\left(\left(1-\frac{7}{n+7}\right)^{6n}\right)}$$

$$= e^{6n\ln\left(1-\frac{7}{n+7}\right)}$$

$$6n\ln\left(1-\frac{7}{n+7}\right)$$

$$= 6 \cdot \frac{\ln\left(1-\frac{7}{n+7}\right)}{1/n}$$

$$6 \lim_{n\to\infty} \frac{\ln\left(1-\frac{7}{n+7}\right)}{1/n}$$

$$\overset{LH}{=} 6 \cdot \lim_{n\to\infty} \frac{\frac{1}{1-7/n+7} \cdot -7 \cdot -\frac{1}{(n+7)^2}}{-\frac{1}{n^2}}$$

$$= -42 \lim_{n\to\infty} \frac{n^2}{(n+7)^2-7\cdot(n+7)}$$

$$\overset{LH}{=} -42 \cdot \lim_{n\to\infty} \frac{2n}{2(n+7)-7}$$

$$\overset{LH}{=} -42 \cdot \lim_{n\to\infty} \frac{2}{2} = -42$$

Final answer: e^{-42}