Name:

Math 116 Practice Final

Directions: WRITE YOUR NAME ON THIS EXAM! Except where indicated, merely finding the answer to a problem is not enough to receive full credit; you must show how you arrived at that answer. DO NOT convert irrational numbers such as $\sqrt{3}$ or π into decimal approximations; just leave them as they are.

1) a) What are the rectangular (Cartesian) coordinates of the polar point $\left(-8, -\dfrac{\pi}{3}\right)$?

b) What is a representation in polar coordinates of the rectangular (Cartesian) point $(0, 15)$?

2) Calculate the first derivative of the following functions:

a) $x^3 \ln(8x^2)$

b) $\arctan(e^{2x})$

3) Determine the equation of the tangent line to the parametric curve determined by $f(t) = \langle 5t^9 - 11t + 20, \tan(t + \frac{\pi}{4}) \rangle$ at the point $t = 0$.

4) Solve the following non-calculus problems.

a) Find the EXACT value of $\sin(\arccos(21/25))$.

b) After a few months of dry spells, you are offered an investment opportunity! Sergey Brin will give you $120 worth of profit-sharing ad revenue from Google and Jerry Jones offers you $400 worth of ad revenue in billboards plastered around the new Cowboys stadium, both with interest. If the Google revenue accrues interest at the rate of 15% a year and the Cowboys' at 3% a year (both compounded continuously), find which is the better investment after 10 years.

5) a) Set up an equation for the arclength of the parametric curve $f(t) = \langle e^t \cos(t), e^t \sin(t) \rangle$ from $t = 0$ to $t = \ln(8)$.

b) Find the arclength of the portion of the curve described in part a).

6) Evaluate the integral $\displaystyle\int_{-4}^{0} \frac{dx}{x^2 - 2x - 15}$.

7) Find the area enclosed by the inner loop of the polar curve $r = \sqrt{3} + 2\cos(\theta)$. Note the graph below.
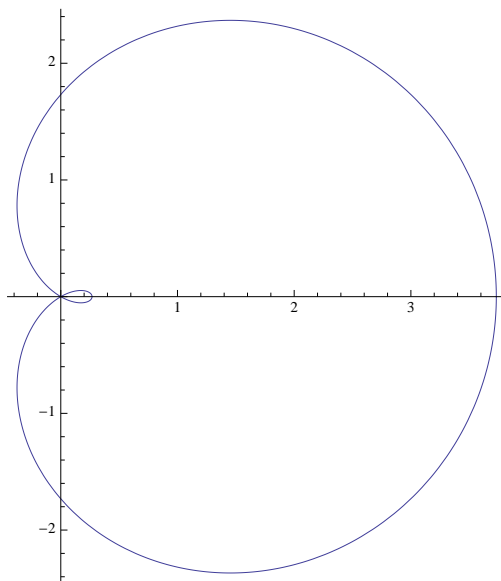
8) Determine the center c of the power series $\displaystyle\sum_{n=2}^{\infty} \frac{(\ln(n))^2(4x-20)^n}{8^n n^2}$.

b) Find the radius of convergence of the power series in part a)

9) Given that the radius of convergence of $\displaystyle\sum_{n=2}^{\infty} \frac{(\ln(n))^2(4x-20)^n}{8^n n^2}$ is equal to 2, determine the interval of convergence. (Just think of this as another series question after you plug the numbers in.)

10) Find the sum of the series or show that it diverges.

a) $\displaystyle\sum_{n=2}^{\infty} \frac{8^{2n+1}}{5^{4n}}$

b) $\displaystyle\sum_{n=0}^{\infty} \frac{(-1)^n 13^{2n}}{n!}$ (*Hint:* Taylor series.)

11) Find the limit of the expression or show that it diverges.

a) $\lim\limits_{n\to\infty} 21n \sin\left(\dfrac{2}{n}\right)$

b) $\lim\limits_{x\to\infty} x^{1/\ln(\sqrt[3]{x})}$