Name:

Math 116 Final

December 21, 2010

Directions: WRITE YOUR NAME ON THIS EXAM! Except where indicated, merely finding the answer to a problem is not enough to receive full credit; you must show how you arrived at that answer. DO NOT convert irrational numbers such as $\sqrt{3}$ or π into decimal approximations; just leave them as they are.

1) True/false. No justification is necessary.

a) The range of $\arctan(x)$ is $(-\pi/2, \pi/2)$.

b) $\lim_{n\to\infty} \left(\frac{n+1}{n}\right)^n = 1$.

c) The improper integral $\int_{-\infty}^{\infty} f(x)\,dx$ is defined as

$$\lim_{a\to\infty} \int_{-a}^{a} f(x)\,dx.$$

d) If $\lim_{n\to\infty} a_n = 0$, then $\sum_{n=1}^{\infty} a_n$ converges.

e) If p is a real number, then $\sum_{n=1}^{\infty} \frac{1}{n^p}$ converges if $p > 1$ and diverges otherwise.

f) For all positive real numbers x, $\ln(52/x) = \ln(52)/\ln(x)$.

g) $\sqrt{x^2 + y^2} = x + y$ for all positive real numbers x and y.

h) $\sin^2(\theta) = \frac{1 + \cos(\theta)}{2}$.

i) The function e^x has MacLaurin series expansion $e^x = \sum_{n=0}^{\infty} \frac{x^n}{n!}$.

j) The area inside a polar curve $r = f(\theta)$ from $\theta = \theta_0$ to $\theta = \theta_1$ is

$$A = \frac{1}{2} \int_{\theta_0}^{\theta_1} [f'(\theta)]^2 \, d\theta.$$

2) a) (6 points) What are the rectangular (Cartesian) coordinates of the polar point $\left(-12, -\dfrac{7\pi}{6}\right)$?

b) (6 points) What is a representation in polar coordinates of the rectangular (Cartesian) point $(-2, 2)$?

3) Calculate the first derivative of the following functions:

a) (8 points) $\ln((3x)^{4x^2})$, $x > 0$

b) (8 points) $e^{\arctan(\pi x)}$

4) Consider the parametric curve determined by $f(t) = \left\langle 10^t, \dfrac{2}{1+t^2} \right\rangle$.

a) (2 points) Find the value of t for which $f(t) = \langle 10, 1 \rangle$

b) (10 points) Determine the equation of the tangent line to the curve at the point $(10, 1)$.

5) Solve the following non-calculus problems.

a) (8 points) Find the EXACT value of tan(arcsin(5/32)). NO DECIMALS!

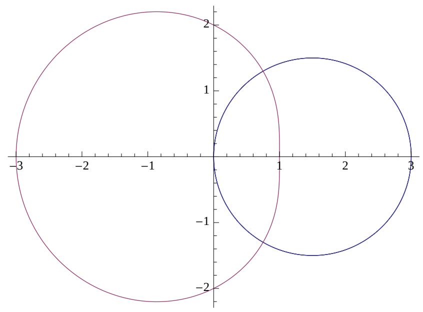
b) (8 points) You are searching for mortgage plans to help you purchase a brand-new home. Frenemy Bank will finance you for $200,000 on a 30 year mortgage at 3.5% interest, compounded continuously. The Bank of Impossible Secrets offers $250,000 over the same timeframe at 2.5% interest, again compounded continuously. Both banks assure you that you will be better off paying the amount owed in full after 30 years instead of using monthly installments. Assuming you are gullible enough to select one and only one of these offers, which will cost you more at the end of 30 years?

6) a) (7 points) Set up an equation for the arclength of the parametric curve $f(t) = \langle \ln(\sec(t) + \tan(t)), \sec(t) \rangle$ from $t = 0$ to $t = \dfrac{\pi}{4}$.

b) (10 points) Find the arclength of the portion of the curve described in part a).

7) (16 points) Evaluate the integral $\displaystyle\int_1^5 \frac{dx}{2x^3+6x^2-36x}$.

8) Consider the region in the 1st quadrant inside both the curves $r = 3\cos(\theta)$ and $r = 2 - \cos(\theta)$. See the graph below.



a) (2 points) Shade the indicated region.

b) (4 points) Find the θ value for which the curves intersect if $0 \leq \theta \leq \dfrac{\pi}{2}$.

c) (8 points) Set up BUT DO NOT EVALUATE an integral or integrals representing the area of the indicated region.

9) Consider the power series $\displaystyle\sum_{n=3}^{\infty} \frac{5^{2n}(1-2x)^n}{n^{3/2}}$.

a) (4 points) What is the center of the series?

b) (12 points) Find the radius of convergence of the series.

10) a) (2 points) Does the series $\displaystyle\sum_{n=1}^{\infty} \frac{2^{3n}}{3^{n+2}}$ converge? Just answer "yes" or "no," no work required.

b) (5 points) Justify your reasoning from part a).

c) (8 points) Find the value of the series $\displaystyle\sum_{n=1}^{\infty} \frac{3^{n+2}}{2^{3n}}$ or show that it diverges.

11) Determine whether the following series converge or diverge. CLEARLY STATE WHICH TEST YOU ARE USING.

a) (10 points) $\displaystyle\sum_{n=1}^{\infty} \frac{\arctan(n)}{(n+2)!}$

b) (8 points) $\displaystyle\sum_{n=121}^{\infty} \frac{\sqrt{16n^2+9n+1}}{2n-3}$

12) a) (8 points) Calculate the limit of the expression

$$\lim_{x\to\infty}\left(\frac{x^2}{2e^{2x}} + xe^{-2x}\right).$$

or show that it diverges.

b) (10 points) Evaluate the integral

$$\int_3^\infty e^{-2x}(x^2 + x - 1)\ dx.$$

Hint: Simplify the antiderivative and use part a).

13) (10 points) Determine whether

$$\sum_{n=3}^{\infty} \frac{\sin^2\left(\frac{\pi}{n}\right)}{n^2}$$

converges or diverges. CLEARLY STATE WHICH TEST YOU ARE USING.

Bonus Question One: (10 points) If $a_1 = 3$ and

$$a_n = \sqrt{3 + a_{n-1}}$$

for $n \geq 2$, find $\lim_{n\to\infty} a_n$. You may assume the limit exists (i.e., I am telling you that the sequence converges and I want you to tell me what exactly it converges to).

Bonus Question Two: (10 points) Evaluate

$$\lim_{x \to 0} \frac{1}{5x^4} \int_0^x \sin(t^3)\ dt.$$