Name:

Math 116 Final

December 16, 2014

Directions: WRITE YOUR NAME ON THIS EXAM! Except where indicated, merely finding the answer to a problem is not enough to receive full credit; you must show how you arrived at that answer. Decimal approximations to symbolic quanitities, accurate to four decimal places, are acceptable.

Now, if you had a choice on whether to take this exam, please indicate your understanding of the potential consequences by signing the statement below:

I understand that by taking this exam, I may lower my grade from what it was before the final.

Signed: _____

1) (10 points, 2 points each) Partial fractions question.

2) (10 points, 2 points each) Fill in the blank.

a) The graph of the polar curve $r = 3$ is a _____.

b) The domain of $f(x) = \arcsin(x)$ is _____.

c) The sum of a convergent series is _____.

d) $\sum_{n=1}^{\infty} \frac{1}{n^p}$ converges for all values of p that are _____.

e) When integrating $\cos^2(\theta)$, use the half angle formula $\cos^2(\theta) =$ _____.

3) LIKE THIS a) (6 points) What are the rectangular (Cartesian) coordinates of the polar point $\left(21, \frac{9\pi}{2}\right)$?

b) (6 points) What are two representations in polar coordinates of the rectangular (Cartesian) point $(-88, 88)$?

4) LIKE THIS Consider the parametric curve defined by $f(t) = \langle 18^{12t}, \arctan(\tan(3\pi t)) \rangle$.

a) (3 points) Find the value of t for which $f(t) = \langle 5832, -\pi/4 \rangle$.

b) (10 points) Determine the equation of the tangent line to the curve at the point $\langle 5832, -\pi/4 \rangle$.

5) LIKE THIS Let $f(t) = \left\langle \frac{4\sqrt{5}}{3}t^{3/2}, t^2/2 - 5t + 11 \right\rangle$.

a) (6 points) Set up an equation for the arclength of the graph of f from $t = 2$ to $t = 6$.

b) (10 points) Determine the arclength specified in part a).

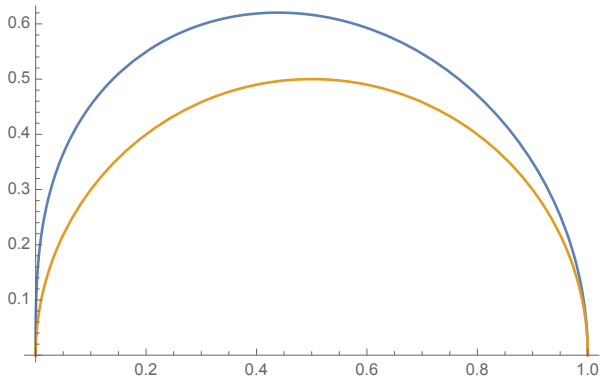
6) LIKE THIS Consider the power series $\sum_{n=3}^{\infty} \frac{(3x+6)^n \sqrt{n}}{2^n}$.

a) (3 points) What is the center of the series?

b) (12 points) Find the radius of convergence of the series.

7) (12 points) Laplace Transform questions

8) The graphs of the circle $r = \sqrt{\cos(\theta)}$ and $r = \cos(\theta)$ in the first quadrant are shown below.



a) (3 points) Shade the region inside $r = \sqrt{\cos(\theta)}$ but outside $r = \cos(\theta)$ in the first quadrant.

b) (6 points) Find the θ values of the intersection points of $r = \sqrt{\cos(\theta)}$ and $r = \cos(\theta)$ in the first quadrant.

c) (12 points) Find the area inside $r = \sqrt{\cos(\theta)}$ but outside $r = \cos(\theta)$ in the first quadrant.

9) (15 points) Power series satisies differential equation (same as exam 2, the other linearly independent solution)

10) (12 points) Newton's Law of Cooling?

11) Brine Problem

12) LIKE THIS Find the sum of the series or show that it diverges.

a) (12 points) $\displaystyle\sum_{n=2}^{\infty}\left(\frac{4^{2n}}{3^{3n+1}}\right)$

b) (12 points) $\displaystyle\sum_{n=0}^{\infty}\frac{(-1)^n\pi^{2n}}{9^n(2n)!}$

13) LIKE THIS Calculate the following limits.

a) (8 points) $\lim_{n \to 0^+} \dfrac{\ln(n) + 4n}{\ln((2n)^3)}$

b) (12 points) $\lim_{x \to \infty} \left(1 - \dfrac{1}{x^3}\right)^{2x^3}$

Bonus Question One: (10 points) Given that $\sum_{n=1}^{\infty} \frac{1}{n^2} = \frac{\pi^2}{6}$, calculate $\sum_{n=0}^{\infty} \frac{1}{(2n+1)^2}$.

Bonus Question Two: (10 points) A circle of radius 1 is tangent to both rays determining the graph of $y = 2|x|$. What is the y-coordinate of the center of the circle?