Name:

Math 116 Final

December 21, 2016

Directions: WRITE YOUR NAME ON THIS EXAM! Except where indicated, merely finding the answer to a problem is not enough to receive full credit; you must show how you arrived at that answer. Decimal approximations to symbolic quanitities, accurate to four decimal places, are acceptable.

Now, if you had a choice on whether to take this exam, please indicate your understanding of the potential consequences by signing the statement below:

I understand that by taking this exam, I may lower my grade from what it was before the final.

Signed: ____________________________

1) a) (6 points) What are the rectangular (Cartesian) coordinates of the polar point $(-18, 3\pi)$?

b) (6 points) What are two representations in polar coordinates of the rectangular (Cartesian) point $(3\sqrt{3}, -3)$?

2) (12 points) Recall that the area inside a polar curve $r = f(\theta)$ from $\theta = \theta_0$ to $\theta = \theta_1$ is given by

$$A = \frac{1}{2}\int_{\theta_0}^{\theta_1} (f(\theta))^2 \, d\theta.$$

Use this formula to compute the area inside $r = \cos(2\theta)$ from $\theta = -\pi/4$ to $\theta = 3\pi/4$.

2) Consider the parametric curve defined by $f(t) = \langle t^{-t}, \ln(t/2) \rangle$.

a) (3 points) Find the value of t for which $f(t) = \langle 1/4, 0 \rangle$.

b) (12 points) Determine the equation of the tangent line to the curve at the point $\langle 1/4, 0 \rangle$.

4) Let $f(t) = \left\langle \frac{4t^{3/2}}{3} + 1, \frac{t^2}{2} - t + 6 \right\rangle$.

a) (7 points) Set up an equation for the arclength of the graph of f from $t = 2$ to $t = 4$.

b) (10 points) Determine the arclength specified in part a).

5) Consider the power series $\sum_{n=1}^{\infty} \frac{(4-2x)^n}{3n+5}$.

a) (3 points) What is the center of the series?

b) (12 points) Find the radius of convergence of the series.

c) (2 points) Given that the radius of convergence is 1/2, list two numbers other than the center for which the series converges.

6) a) (3 points) If f is continuous on $[0, \infty)$, define $\int_0^\infty f(t)\, dt$.

b) (7 points) State L'Hôpital's Rule.

c) (20 points) Compute the Laplace Transform of $f(t) = t + 1$. Recall that the Laplace Transform of a function f is defined as

$$\mathcal{L}\{f\}(w) = \int_0^\infty f(t) e^{-wt}\, dt.$$

7) a) (6 points) Define what it means to "add up" the terms of an infinite sequence of real numbers $(a_n)_{n=1}^{\infty}$.

b) (10 points) Either find the sum of

$$\sum_{n=1}^{\infty} \left(\frac{1}{\sqrt{n}} - \frac{1}{\sqrt{n+1}} \right)$$

or show that the series diverges.

8) (15 points) Let $f(x) = \sum_{n=0}^{\infty} \frac{(-3)^n x^n}{n!}$. Show that f satisfies the differential equation

$$y'' + 6y' + 9y = 0.$$

9) A mixture of water and salt flows into a tank containing 500L of water in which 4kg of salt are initially dissolved. Let $x(t)$ denote the amount of salt in the tank at time t. Suppose the mixture flows into the tank at 10L/min and flows out at the same rate.

a) (2 points) Determine the initial condition on x.

b) (8 points) Find an equation for $\frac{dx}{dt}$ in terms of $x(t)$ if the mixture flowing in contains .2kg/L of sugar.

c) (12 points) Find an explicit formula for the amount of salt $x(t)$ in the tank, i.e., solve for $x(t)$.

10) a) (11 points) Find the partial fraction decomposition of $\dfrac{x+2}{(x+1)(x^2+1)}$.

b) (8 points) Compute the value of $\displaystyle\int_0^1 \frac{x+2}{(x+1)(x^2+1)}\,dx$.

11) a) (8 points) Find the sum of the series $\sum_{n=3}^{\infty} \frac{5^n}{2^{3n}}$.

b) (10 points) Compute $\lim_{x\to\infty} \left(1 - \frac{2016}{x}\right)^x$.