Name:

Math 116 Final

December 18, 2017

Directions: WRITE YOUR NAME ON THIS EXAM! Except where indicated, merely finding the answer to a problem is not enough to receive full credit; you must show how you arrived at that answer. Decimal approximations to symbolic quanitities, accurate to four decimal places, are acceptable.

1) Fill in the blank.

a) (1 point) The number of representations a Cartesian point (x, y) has in polar coordinates is __________.

b) (3 points) When integrating $\cos^2(\theta)$, use the half angle formula $\cos^2(\theta) =$ _______________.

c) (1 point) The graph of the polar curve $r = 12$ is a __________.

d) (3 points) The geometric series $\sum_{n=1}^{\infty} x^n$ converges for all values of x such that __________.

e) (1 point) The MacLaurin series $\sum_{n=0}^{\infty} \frac{x^n}{n!}$ represents the function __________.

2) a) (6 points) Given that the radius of convergence of a power series is 6 and its center is -11, list

(i) two numbers other than the center for which the series definitely converges.

(ii) two numbers where the series definitely diverges

(iii) the only two numbers where it is unclear whether the series converges or diverges.

b) (1 point) True or false: $\sqrt{16+1} = 4 + 1 = 5$.

c) (1 point) True or false: you need a quotient to apply l'Hopital's rule.

3) a) (6 points) What are the rectangular (Cartesian) coordinates of the polar point $(-10, -3\pi/4)$?

b) (6 points) What are two representations in polar coordinates of the rectangular (Cartesian) point $(-5\sqrt{3}, 5)$?

4) a) (3 points) Parameterize $y = f(x)$.

b) (9 points) Compute the curvature of the graph of $f(x) = 4^{x^3}$ at the point $x = 1$. Recall that the curvature for a parametric cuve $\langle x(t), y(t) \rangle$ is given by

$$\kappa(t) = \frac{x'(t)y''(t) - y'(t)x''(t)}{((x'(t))^2 + (y'(t))^2)^{3/2}}.$$

5) (14 points) Consider the parametric curve defined by $f(t) = \langle e^{-\ln(5/t)}, (2t)^t \rangle$. Determine the equation of the tangent line to the curve at the point $t = 5$.

6) Let $f(t) = \left\langle \dfrac{4t^{3/2}}{3} + 1, \dfrac{t^2}{2} - t + 6 \right\rangle$.

a) (8 points) Set up an equation for the surface area obtained by revolving the graph of f about the y-axis from $t = 1$ to $t = 2$.

b) (13 points) Determine the surface area specified in part a).

7) Consider the power series $\displaystyle\sum_{n=7}^{\infty} \frac{n(20-4x)^n}{8^n}$.

a) (3 points) What is the center of the series?

b) (12 points) Find the radius of convergence of the series.

8) a) (8 points) Find the partial fraction decomposition of $\dfrac{x+5}{(x+6)(3-x)}$.

b) (6 points) Compute the value of $\displaystyle\int_0^1 \frac{x+5}{(x+6)(3-x)}\,dx$.

9) a) (3 points) If f is continuous on $[0, \infty)$, define $\displaystyle\int_0^\infty f(t)\,dt$.

c) (20 points) Compute the Laplace Transform of $f(t) = (t-2)^2$. Recall that the Laplace Transform of a function f is defined as

$$\mathcal{L}\{f\}(s) = \int_0^\infty f(t)e^{-st}\,dt.$$

c) (3 points) For what values of s is the Laplace Transform of $f(t) = (t-2)^2$ defined?

10) (15 points) Let $f(x) = \sum_{n=0}^{\infty} \frac{(-16)^n x^{2n+1}}{(2n+1)!}$. Show that f satisfies the differential equation

$$y'' + 16y = 0.$$

11) a) (6 points) Define what it means to "add up" the terms of an infinite sequence of real numbers $(a_n)_{n=1}^{\infty}$.

b) (10 points) Either find the sum of

$$\sum_{n=4}^{\infty}(\arctan(n-3) - \arctan(n-2))$$

or show that the series diverges.

12) Your 200°F cup of coffee is left in a 72°F room. After 2 minutes, you check the temperature of your coffee and find it to be a still-scalding 190°F.

a) (15 points) Find an explicit formula for the temperature $f(t)$ of the coffee.

b) (5 points) If you want your coffee to have cooled to 120°F before you begin drinking it, how long will you have to wait after checking the temperature after 2 minutes?

13) a) (8 points) Find the sum of the series $\sum_{n=3}^{\infty} \frac{(-6)^n}{3^{2n+1}}$.

b) (10 points) Compute $\lim_{x\to\infty} \left(\frac{\sqrt{x}}{\sqrt{x}+5}\right)^{\sqrt{9x}}$.