

Name:

Math 116 Final

April 25, 2011

Directions: WRITE YOUR NAME ON THIS EXAM! Except where indicated, merely finding the answer to a problem is not enough to receive full credit; you must show how you arrived at that answer. DO NOT convert irrational numbers such as $\sqrt{3}$ or π into decimal approximations; just leave them as they are.

- 1) a) (6 points) What are the rectangular (Cartesian) coordinates of the polar point $\left(17, -\frac{5\pi}{3}\right)$?
- b) (6 points) What is a representation in polar coordinates of the rectangular (Cartesian) point $(\sqrt{12}, 2)$?

2) Calculate the first derivative of the following functions:

a) (8 points) $f(x) = \arctan(\ln(x)), x > 0$

b) (8 points) $g(x) = xe^{-x^2}$

3) Consider the parametric curve determined by

$$f(t) = \left\langle t + \cos\left(\frac{\pi t}{2}\right), \ln(1 - t) \right\rangle.$$

a) (2 points) Find the value of t for which $f(t) = \langle 1, 0 \rangle$

b) (10 points) Determine the equation of the tangent line to the curve at the point $(1, 0)$. SIMPLIFY ALL NUMBERS IN YOUR ANSWER!

4) Solve the following non-calculus problem.

Desperate for outside capital, Fukushima Power, Inc. is offering you the chance to purchase \$2000 worth of “hot” stock in their company with a guarantee of 10% interest accrued, compounded continuously. Jealous of Fukushima’s news coverage, BP gives you the opportunity to purchase \$1500 of their “crude” options, with a guaranteed 12% rate, again compounded continuously.

a) (8 points) After 8 years, which of the accounts would have the most money in it?

b) (4 points) After 8 years, which account gives a better return for your investment, reflected as a multiple of the principal?

5) Consider the power series $\sum_{n=3}^{\infty} \frac{n^{5/2}(7-3x)^n}{4^n}$.

a) (4 points) What is the center of the series?

b) (12 points) Find the radius of convergence of the series.

6) (12 points) Given that the radius of convergence of $\sum_{n=3}^{\infty} \frac{n^{5/2}(7-3x)^n}{4^n}$ is equal to $4/3$, determine the interval of convergence.

7) Determine whether the following series converge or diverge. CLEARLY STATE WHICH TEST YOU ARE USING.

a) (8 points) $\sum_{n=2}^{\infty} \frac{(-1)^n \ln(n)}{n^{9/11}}$

b) (10 points) $\sum_{n=2}^{\infty} \frac{1}{n^2 + 4}$

8) The following series converge. Find a sufficient number of terms to estimate the sum to within .02. Be sure to justify your answer with calculations.

a) (6 points) $\sum_{n=2}^{\infty} \frac{(-1)^n \ln(n)}{n^{9/11}}$

b) (12 points) $\sum_{n=2}^{\infty} \frac{1}{n^2 + 4}$

9) Find the sum of the series or show that it diverges.

a) (12 points) $\sum_{n=3}^{\infty} \frac{13^{2n+1}}{7^{3n}}$

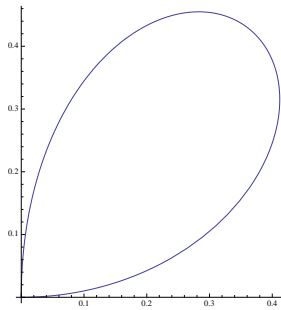
b) (14 points) $\sum_{n=1}^{\infty} \frac{1}{n^2 + 7n + 12}$ (*Hint: two kinds of partials*)

10) Calculate the following limits.

a) (10 points) $\lim_{x \rightarrow 0} \frac{x^2 + x - \sin(x)}{1 - \cos(6x)}$

b) (10 points) $\lim_{n \rightarrow \infty} \left(1 + \frac{10}{n}\right)^n$

11) Consider the region in the 1st quadrant inside the curve $r = \theta \cos(\theta)$. See the graph below.



- a) (4 points) Find the θ values which determine the region.
- c) (14 points) Calculate the area of the given region. (*Hint*: look at the fill-in-the-blank and remember what you were promised!)