

Name:

## Math 116 Final

April 25, 2012

**Directions:** WRITE YOUR NAME ON THIS EXAM! Except where indicated, merely finding the answer to a problem is not enough to receive full credit; you must show how you arrived at that answer. DO NOT convert irrational numbers such as  $\sqrt{3}$  or  $\pi$  into decimal approximations; just leave them as they are.

- 1) a) (6 points) What are the rectangular (Cartesian) coordinates of the polar point  $\left(-25, \frac{7\pi}{4}\right)$ ?
- b) (6 points) What is a representation in polar coordinates of the rectangular (Cartesian) point  $(\sqrt{18}, -\sqrt{54})$ ?

**2)** Consider the parametric curve defined by  $x(t) = \arctan(\sqrt{t})$ ,  $y(t) = \ln(e^{t^3})$ .

a) (3 points) Find the value of  $t$  for which  $x(t) = \frac{\pi}{4}$  and  $y(t) = 1$

b) (10 points) Determine the equation of the tangent line to the curve at the point  $\left(\frac{\pi}{4}, 1\right)$ . SIMPLIFY ALL NUMBERS IN YOUR ANSWER!

3) Solve the following non-calculus problems.

a) (8 points) Find the EXACT value of  $\cos(\arctan(1/9))$ . NO DECIMALS.

b) (8 points) Lucas Papademos, interim Prime Minister of Greece, is tired of austerity measures. To help bolster Greece's economy, he is offering you stock in Lambda Olive Oil. To sweeten the deal, he will compound your initial investment continuously at a rate of 5%. In 8 years, when Greece will still be burdened with crushing debt, how much of their money will you have gained from an initial investment of \$1000? Round to the nearest penny.

4) Consider the power series  $\sum_{n=7}^{\infty} \frac{(9x - 1)^n}{n^{10}6^{2n}}$ .

a) (4 points) What is the center of the series?

b) (12 points) Find the radius of convergence of the series.

5) (12 points) Given that the radius of convergence of  $\sum_{n=7}^{\infty} \frac{(9x - 1)^n}{n^{10}6^{2n}}$  is equal to 4, determine the interval of convergence.

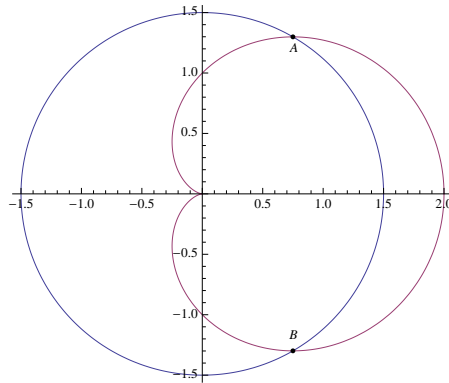
**6)** (10 points) Evaluate the indefinite integral  $\int x \cos(x) dx$ .

7) Evaluate the following definite integrals.

a) (12 points)  $\int_{\pi/12}^{\pi/8} \sin^2(4x) dx.$

b) (12 points)  $\int_0^{\infty} \frac{1}{1 + 25x^2} dx$

8) The graphs of the circle  $r = 3/2$  and the cardioid  $r = 1 + \cos(\theta)$  are shown in the  $xy$ -plane below.



- a) (3 points) Shade the region inside the circle but outside the cardioid.
- b) (6 points) Find the  $\theta$  values of the intersection points  $A$  and  $B$ .
- c) (6 points) Set up the definite integral(s) needed to find the area of the region inside the circle but outside the cardioid. Do not evaluate the integral(s).



9) Calculate the following limits.

a) (8 points)  $\lim_{n \rightarrow 3} \frac{\sin(\pi n)}{n^2 - 9}$

b) (12 points)  $\lim_{x \rightarrow \infty} (1 + e^{-x})^{e^x}$

10) Find the sum of the series or show that it diverges.

a) (12 points)  $\sum_{n=3}^{\infty} \left( \cos\left(\frac{\pi}{n}\right) - \cos\left(\frac{\pi}{n+1}\right) \right)$

b) (12 points)  $\sum_{n=1}^{\infty} \frac{\pi^{2n+1}}{n!}$

**11)** Let  $f(x) = \ln(1 - x^2)$ .

a) (6 points) Set up an equation for the arclength of the graph of  $f$  from  $x = 0$  to  $x = 1/4$ .

b) (12 points) Determine the arclength specified in part a). (*Hint:* partial fractions, but be careful...)