Name:

Math 116 Final

April 25, 2018

Directions:

1. WRITE YOUR NAME ON THIS TEST!
2. Except where indicated, merely finding the answer to a problem is not enough to receive full credit; you must show how you arrived at that answer.
3. Unless otherwise indicated, decimal approximations for a numerical answer accurate to 4 decimal places are acceptable.
4. If you have a question, raise your hand or come up and ask me.

1) Fill in the blank.

a) (1 point) The value of the limit $\lim_{x \to \infty} e^{-x}$ is equal to __________.

b) The geometric series $\sum_{n=0}^{\infty} x^n$ converges for all values of x such that __________.

c) (1 point) The number of representations a Cartesian point (x, y) has in polar coordinates is __________.

d) (3 points) (3 points) The formula for the surface area of a parametric curve $x = x(t)$, $y = y(t)$ revolved about the x-axis from $t = a$ to $t = b$ is

$$\int_a^b 2\pi \underline{\hspace{3em}} \sqrt{(\underline{\hspace{3em}})^2 + (\underline{\hspace{3em}})^2}\, dt.$$

e) (1 point) The MacLaurin series $\sum_{n=0}^{\infty} \frac{(-1)^n x^{2n}}{(2n)!}$ represents the function __________.

2) a) (6 points) Given that the radius of convergence of a power series is 4 and its center is -2, list

(i) two numbers other than the center for which the series definitely converges.

(ii) two numbers where the series definitely diverges

(iii) the only two numbers where it is unclear whether the series converges or diverges.

b) (1 point) True or false: for all values of x, $\sqrt{9+x^2} = x + 3$.

c) (1 point) True or false: you don't need a quotient to apply l'Hopital's rule.

3) a) (6 points) What are the rectangular (Cartesian) coordinates of the polar point $(8, -5\pi/6)$?

b) (9 points) What are two representations in polar coordinates of the rectangular (Cartesian) point $(-3, -3)$?

4) a) (2 points) Parameterize $y = f(x)$.

b) (10 points) Compute the curvature of the graph of $f(x) = e^{x^2}$ at the point $(0, 1)$. Recall that the curvature for a parametric curve $x = x(t)$, $y = y(t)$ is given by

$$\kappa(t) = \frac{x'(t)y''(t) - y'(t)x''(t)}{((x'(t))^2 + (y'(t))^2)^{3/2}}.$$

5) (14 points) Consider the parametric curve defined by

$$x(t) = \ln(3t), \;\; y(t) = t^{\sqrt{t}}.$$

Determine the equation of the tangent line to the curve at the point $t = 1$.

6) a) (7 points) Set up an equation for the arclength of the parametric curve $x(t) = \ln(\cos(t))$, $y(t) = t$ from $t = 0$ to $t = \frac{\pi}{6}$.

b) (5 points) Find the arclength of the portion of the curve described in part a).

7) Consider the power series $\displaystyle\sum_{n=3}^{\infty} \frac{(9-18x)^n}{(n+2)^2 45^n}$.

a) (2 points) What is the center of the series?

b) (15 points) Find the radius of convergence of the series.

8) (8 points) Compute the value of $\displaystyle\int_0^{\pi/2} \frac{\cos(x)}{\sin^2(x)+1}\, dx.$

9) a) (3 points) If f is continuous on $[0, \infty)$, define $\int_0^\infty f(t)\, dt$.

b) (16 points) Compute the Laplace Transform of $f(t) = 6t + 1$. Recall that the Laplace Transform of a function f is defined as

$$\mathcal{L}\{f\}(s) = \int_0^\infty f(t)e^{-st}\, dt.$$

c) (2 points) For what values of s is the Laplace Transform of $f(t) = 6t+1$ defined?

10) (10 points) Let $f(x) = \displaystyle\sum_{n=0}^{\infty} \frac{(-8)^{n+1}x^n}{n!}$. Show that f satisfies the differential equation

$$y' + 8y = 0.$$

11) a) (6 points) Define what it means to “add up” the terms of an infinite sequence of real numbers $(a_n)_{n=1}^{\infty}$.

b) (19 points) Either find the sum of

$$\sum_{n=2}^{\infty} \left(\frac{1}{n^2 + 5n + 6} \right)$$

or show that the series diverges.

12) A tank contains 900L of water with 6 kg of dissolved fructose initially present. A mixture containing water with .01 kg/L of fructose flows into the tank at a rate of 8L/min and flows out at the same rate. If you are pedantic, the mixture is kept uniform by stirring. Let $x(t)$ denote the amount of sugar in the tank at time t, in kilograms.

a) (10 points) Find an equation for $\frac{dx}{dt}$ in terms of $x(t)$, plugging in all relevant numbers.

b) (15 points) Solve the equation you found in part a) for $x(t)$.

c) (2 points) Find the amount of fructose in the tank after 12 minutes.

13) a) (10 points) Find the sum of the series $\sum_{n=1}^{\infty} \frac{(-99)^{n-1}}{(-10)^{2n}}$.

b) (12 points) Compute $\lim_{x \to \infty} \left(\frac{x^3 + 4}{x^3} \right)^{(2x)^3}$.