Name:

Math 116 Final

Winter 2020 Corona Virus Edition

Directions:

1. You may use your notes, textbook, and a calculator for this exam, but NO OTHER RESOURCES; if I can determine you're cheating on this exam, you'll get a zero.
2. Except where indicated, merely finding the answer to a problem is not enough to receive full credit; you must show how you arrived at that answer.
3. Unless otherwise indicated, DO NOT convert irrational numbers such as $\sqrt{3}$ or π into decimal approximations; just leave them as they are.
4. Once you submit this exam through Canvas, it will factor into your grade in the manner described in the syllabus. NO TAKE-BACKS. Please indicate your understanding of the potential consequences of taking this exam by signing the statement below:

   I understand that by taking this exam, I may lower my grade from what it was before the final.

   Signed: _____

1) a) What are the rectangular (Cartesian) coordinates of the polar point $(13, -5\pi/3)$?

b) What are two representations in polar coordinates of the rectangular (Cartesian) point $(0, -15)$?

2) For this problem, consider the parametric curve given by

$$x(t) = t^3,\ y(t) = \ln(-\cos(\pi t)) + t^2.$$

a) Find the equation of the tangent line to the curve at the point $(-1, 1)$.

b) Find the arc length from $x = 0$ to $x = 2$ of the tangent line to the curve described in part a).

3) A tank contains 1500L of water with 2 kg of dissolved fructose initially present. A mixture containing water with .5 kg/L of fructose flows into the tank at a rate of 20L/min and flows out at the same rate. If you are pedantic, the mixture is kept uniform by stirring. Let $x(t)$ denote the amount of sugar in the tank at time t, in kilograms.

a) Find an equation for $\dfrac{dx}{dt}$ in terms of $x(t)$, plugging in all relevant numbers.

b) Solve the equation you found in part a) for $x(t)$.

c) Find the amount of fructose in the tank after 15 minutes.

When solving this problem, you should:

(i) Show work for evaluating any integrals.

(ii) Use reason to determine whether the answer you obtain at the end is correct.

4) a) Compute the Laplace Transform of $f(t) = t^2 \cdot 9^t$. Recall that the Laplace Transform of a function f is defined as

$$\mathcal{L}\{f\}(w) = \int_0^\infty f(t)e^{-wt}\, dt.$$

b) For what values of w is the Laplace Transform of $f(t) = t^2 \cdot 9^t$ defined?

When solving this problem, you should:

(i) Justify any integrals or limits you have to take with work.

(ii) Support your answer in b) with reasons.

5) Let $f(x) = \sum_{n=1}^{\infty} \frac{n \cdot x^{n-1}}{2^{n-1}}$.

a) Find the radius of convergence of f.

b) Show that $y = f(x)$ satisfies the differential equation

$$2y - 2y' + xy' = 0.$$

c) Find $f(1)$ explicitly. *Hint:* start with a familiar series.

When solving this problem, you should:

(i) Justify any limits you have to take with work.

(ii) Answer part c) explicitly (not with decimals) and provide justification for your answer.

6) Determine, to the best of your ability, the value of

$$\int_0^{2\pi} \frac{1}{5 - 4\cos(\theta)}\, d\theta.$$

Answers without work will count for nothing.

When solving this problem, you should:

(i) Try your best!