

# Math 116 Final

April 29, 2022

**Directions:**

1. WRITE YOUR NAME ON THIS TEST!
2. Except where indicated, merely finding the answer to a problem is not enough to receive full credit; you must show how you arrived at that answer.
3. Unless otherwise indicated, decimal approximations for a numerical answer accurate to 4 decimal places are acceptable.
4. If you have a question, raise your hand or come up and ask me.
5. Now, if you had a choice on whether to take this exam, please indicate your understanding of the potential consequences by signing the statement(s) below:

I understand that by taking this exam, I may lower my grade from what it was before the final.

**Signed:** \_\_\_\_\_

I had a choice on whether to take this exam, and now that I've seen it, I want to keep my pre-final grade!

**Signed:** \_\_\_\_\_

1. a) Given that the radius of convergence of a power series is 2 and its center is  $-4$ , list
- (i) two numbers other than the center for which the series definitely converges,
  - (ii) two numbers where the series definitely diverges, and
  - (iii) the only two numbers where it is unclear whether the series converges or diverges.
- b) The geometric series  $\sum_{n=0}^{\infty} x^n$  converges for all values of  $x$  such that \_\_\_\_\_.
- c) True or false: for all values of  $x$ ,  $\sqrt{100 + x^2} = x + 100$ .
- d) True or false: the series of real numbers  $\sum_{n=1}^{\infty} a_n$  converges if  $\lim_{n \rightarrow \infty} a_n = 0$
- e) For the integral  $\int_0^3 \sqrt{4 + x^2} dx$ , use the trig substitution  $x =$ \_\_\_\_\_.

2. a) What are the rectangular (Cartesian) coordinates of the polar point  $(-10, -7\pi/3)$ ?
- b) What are two representations in polar coordinates of the rectangular (Cartesian) point  $(-18\sqrt{3}, -18)$ ?

3. Consider the parametric curve defined by

$$x(t) = \ln(25/t^2), \quad y(t) = \arctan(t/5).$$

Determine the equation of the tangent line to the curve at the point  $(0, \pi/4)$ .

4. Consider the power series  $\sum_{n=1}^{\infty} \frac{(14x - 7)^n}{42^n}$ .

- a) What is the center of the series?
- b) Find the radius of convergence of the series.

5. a) Set up an equation for the arclength of the parametric curve defined by

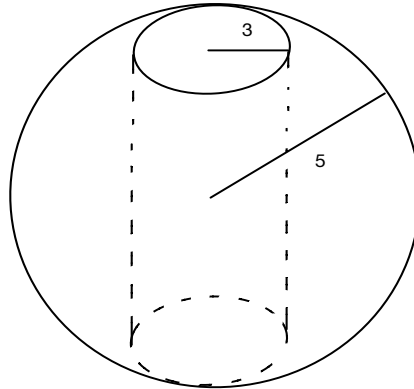
$$x(t) = \cos(t^2), \quad y(t) = \sin(t^2)$$

from  $t = 0$  to  $t = \sqrt{\frac{\pi}{2}}$ .

b) Find the arclength of the portion of the curve described in part a). NOTE: if you don't use calculus, you'd best explain why not!

6. Find the area between the curves  $f(x) = 3x^2$  and  $g(x) = x^2e^x$ .

7. A sphere of radius 5 cm has a hole bored through its center of radius 3 cm. See the picture below.



- a) Determine a formula, using calculus, for the volume remaining after the hole has been bored out.
- b) Find the volume remaining after the hole has been bored out.



8. Recall that the Laplace Transform of a function  $f$  is defined as

$$\mathcal{L}\{f\}(w) = \int_0^{\infty} f(t)e^{-wt} dt.$$

If  $f(t) = e^{2t}$ ,

a) Compute  $\mathcal{L}\{f\}(4)$ .

b) Is  $\mathcal{L}\{f\}(2)$  a real number? Why or why not?

9. Find the function represented by the series

$$\sum_{n=0}^{\infty} (-1)^n \frac{x^{2n+3}}{n!}.$$

10. a) What does it mean to “add up” the terms of an infinite sequence of real numbers  $(a_n)_{n=1}^{\infty}$ ? I am looking for a precise mathematical answer; no philosophy, please!

b) Either find the sum of

$$\sum_{n=1}^{\infty} \left( \frac{1}{n^2 + 13n + 42} \right)$$

or show that the series diverges.

11. A tank contains 1000L of water with 8 kg of dissolved sucrose initially present. A mixture containing water with .02 kg/L of sucrose flows into the tank at a rate of 6L/min and flows out at the same rate. If you are pedantic, the mixture is kept uniform by stirring. Let  $x(t)$  denote the amount of sucrose in the tank at time  $t$ , in kilograms.

a) Find an equation for  $\frac{dx}{dt}$  in terms of  $x(t)$ , plugging in all relevant numbers.

b) Solve the equation you found in part a) for  $x(t)$ .

c) Find the amount of sucrose in the tank after 15 minutes.

12. a) Find the sum of the series  $\sum_{n=2}^{\infty} \frac{(-74)^{n+2}}{4^{3n}}$  or show the series diverges.

b) Compute  $\lim_{n \rightarrow \infty} \left( \frac{n^2}{n^2 - 8} \right)^{5n^2}$ .