Math 116 Final

April 21, 2023

Directions:

1. WRITE YOUR NAME ON THIS TEST!

2. Except where indicated, merely finding the answer to a problem is not enough to receive full credit; you must show how you arrived at that answer.

3. Unless otherwise indicated, decimal approximations for a numerical answer accurate to 4 decimal places are acceptable.

4. If you have a question, raise your hand or come up and ask me.

5. Now, if you had a choice on whether to take this exam, please indicate your understanding of the potential consequences by signing the statement(s) below:

I understand that by taking this exam, I may lower my grade from what it was before the final.

Signed: _____

I had a choice on whether to take this exam, and now that I've seen it, I want to keep my pre-final grade!

Signed: _____

1. a) Given that the radius of convergence of a power series is 1 and its center is -5, list

   (i) two numbers other than the center for which the series definitely converges,

   (ii) two numbers where the series definitely diverges, and

   (iii) the only two numbers where it is unclear whether the series converges or diverges.

b) The geometric series $\sum_{n=0}^{\infty} x^n$ converges for all values of x such that _____.

c) True or false: for all values of x, $(3+x)^2 = 9 + x^2$.

d) True or false: the sum of a convergent series of real numbers is the limit of its partial sums.

e) For the integral $\int_0^3 \sqrt{4 - x^2} \, dx$, use the trig substitution $x=$_____.

2. a) What are the rectangular (Cartesian) coordinates of the polar point $(18, 7\pi/6)$?

   b) What are two representations in polar coordinates of the rectangular (Cartesian) point $(-4, 4)$?

3. Consider the parametric curve defined by

$$x(t) = t \arctan(t - 1), \ y(t) = e^{t^2 - 1}.$$

Determine the equation of the tangent line to the curve at the point $t = 1$.

4. Consider the series $\displaystyle\sum_{n=0}^{\infty} \frac{(x-2)^n}{n^2+1}$.

a) What is the center of the series?

b) Find the radius of convergence of the series.

c) For which value(s) of x does this series converge? In other words, find the interval of convergence of the series. **NOTE**: this part of the question is for assessment purposes only and will not be calculated into your grade. If the radius of convergence you found in b) is finite, the procedure involves using one of the other convergence tests on the two points where the ratio test is inconclusive. See the last page of this exam for a list of these tests.

5. a) Set up an equation for the arclength of the parametric curve defined by

$$x(t) = \ln(6t^2), \ y(t) = \ln(3/t)$$

from $t = 1$ to $t = e$.

b) Find the arclength of the portion of the curve described in part a). NOTE: if you don't use calculus, you'd best explain why not!

6. Consider the parametric curve $\begin{cases} x(t) = -2t \\ y(t) = -t^2 + 1 \end{cases}$. A plot of this curve is provided. Set up **but do not evaluate** an integral which represents the area between the curve and the x-axis.
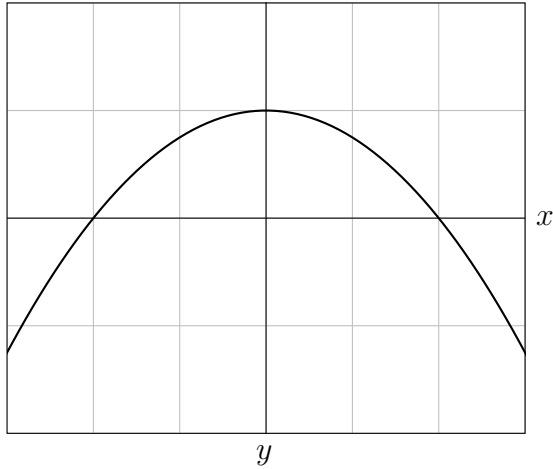
7. Consider the region bounded by the curves $x = 4 - y^2$ and $x = 2 + y^2$. Set up **but do not evaluate** an integral representing the volume of the solid obtained by rotating this region about the y-axis. Be sure to include the limits of integration.

8. Recall that the Laplace Transform of a function f is defined as

$$\mathcal{L}\{f\}(w) = \int_0^\infty f(t)e^{-wt} \, dt.$$

If $f(t) = 2 - 3t$, Compute $\mathcal{L}\{f\}(6)$.

9. Find the function represented by the series

$$\sum_{n=0}^{\infty} (-1)^n \frac{x^{4n+7}}{2n+1}.$$

10. Either find the sum of

$$\sum_{n=1}^{\infty} \frac{12}{n^2 + 3n + 2}$$

or show that the series diverges.

11. A tank contains 1500L of water with 10 kg of dissolved salt initially present. A mixture containing water with .05 kg/L of salt flows into the tank at a rate of 20L/min and flows out at the same rate. If you are pedantic, the mixture is kept uniform by stirring. Let $s(t)$ denote the amount of salt in the tank at time t, in kilograms.

a) What is $s(0)$?

b) Find an equation for $\dfrac{ds}{dt}$ in terms of $s(t)$, plugging in all relevant numbers.

c) Solve the equation you found in part b) for $s(t)$.

d) As time goes to ∞, what value does the amount of salt in the tank get closer to?

12. a) Find the sum of the series $\sum_{n=1}^{\infty} \dfrac{7^{2n+4}}{42^{n-1}}$ or show the series diverges.

b) Compute $\lim\limits_{n\to\infty} \left(\dfrac{\sqrt{n}+3}{\sqrt{n}} \right)^{\sqrt{4n}}$.