W 22 Exam 3

1) a) No -

$a_1 = \frac{5}{8}$

$a_2 = \frac{25}{11}$

$a_3 = \frac{125}{16}$

$\frac{a_2}{a_1} = \frac{11}{40}$

$\frac{a_3}{a_2} = \frac{16}{55} \neq \frac{a_2}{a_1}$

b) yes!

$$a_n = \frac{14^{n-8}}{3^{3n-2}}$$

$$a_{n+1} = \frac{14^{n+1-8}}{3^{3(n+1)-2}}$$

$$= \frac{14^{n-7}}{3^{3n+1}}$$

$$\frac{a_{n+1}}{a_n} = \frac{14^{n-7}}{3^{3n+1}} \cdot \frac{3^{3n-2}}{14^{n-8}}$$

$$= \frac{14^{n-7}}{14^{n-8}} \cdot \frac{3^{3n-2}}{3^{3n+1}}$$

$$= \frac{14}{3^3} = \frac{14}{27} < 1$$

the series converges to

$$\frac{\frac{14^{-4}}{3^{10}}}{1 - \frac{14}{27}}$$

2)

$$S_1 = a_1 = 2 - 2^{\frac{1}{\sqrt{2}}}$$

$$S_2 = a_1 + a_2 = 2 - \cancel{2^{\frac{1}{\sqrt{2}}}} + \cancel{2^{\frac{1}{\sqrt{2}}}} - 2^{\frac{1}{\sqrt{3}}}$$

$$= 2 - 2^{\frac{1}{\sqrt{3}}}$$

$$S_3 = S_2 + a_3 = 2 - \cancel{2^{\frac{1}{\sqrt{3}}}} + \cancel{2^{\frac{1}{\sqrt{3}}}} - 2^{\frac{1}{\sqrt{4}}}$$

$$= 2 - 2^{\frac{1}{\sqrt{4}}}$$

In general, $S_k = 2 - 2^{\frac{1}{\sqrt{k+1}}}$

$$\lim_{k \to \infty} S_k = 2 - 2^0 = 1$$

3) Ratio test:

$$a_{n+1} = \frac{(-17)^{n+1}}{(2(n+1))!}$$

$$= \frac{(-17)^{n+1}}{(2n+2)!}$$

$$\frac{a_{n+1}}{a_n} = \frac{(-17)^{n+1}}{(2n+2)!} \cdot \frac{(2n)!}{(-17)^n}$$

$$= \frac{(-17)^{\cancel{n+1}}}{\cancel{(-17)^n}} \cdot \frac{\cancel{(2n)!}}{(2n+2)(2n+1)\cancel{(2n)!}}$$

$$= \frac{-17}{(2n+2)(2n+1)}$$

$$\left|\frac{a_{n+1}}{a_n}\right| = \frac{17}{(2n+2)(2n+1)}$$

$$\lim_{n\to\infty}\left|\frac{a_{n+1}}{a_n}\right| = \lim_{n\to\infty}\frac{17}{(2n+2)(2n+1)}$$

$$= 0 < 1$$

So the series converges. This says that the sequence converges to zero.