Announcements

1) HW #6 due tomorrow, all webwork. Practice Problems on Canvas.

2) Office hours 11-12

Power Series

(Section 11.8)

Idea: introduce a variable into your series, whose power changes as n changes

Example 1: $1 + x + x^2 + x^3 + \cdots$

$$= \sum_{n=0}^{\infty} x^n$$

Sum of the series depends on x.

$x = \frac{1}{2}$, sum is 2

$x = 0$, sum is 1

$x = 1$, sum diverges

$x = -1$, sum diverges

In general, we get

$$\boxed{\sum_{n=0}^{\infty} x^n = \begin{cases} \text{divergent}, & |x| \geq 1 \\ \dfrac{1}{1-x}, & |x| < 1 \end{cases}}$$

Q: For which x does the series converge?

A: This is the focus of the rest of the class:

For which x does

$$\sum_{n=0}^{\infty} a_n x^n \text{ converge,}$$

where $(a_n)_{n=1}^{\infty}$ is a sequence.

The Center

In general, $\sum_{n=0}^{\infty} a_n x^n$ converges

at least for $\boxed{x=0}$. If

we shift the series and get

$$\sum_{n=0}^{\infty} a_n (x-c)^n \quad \text{for } c \text{ a number}$$

it converges at least for $x=c$.

The number c is called the

center of the power.

Q: How do we tell if there are other points where the series converges?

A: The ratio test!

The Ratio Test

(Section 11.1)

Modeled after the geometric series

- but there, the ratio between consecutive terms is constant.

The ratio test is for when the ratio isn't constant.

Ratio Test (Section 11.6)

Given a power series

$$\sum_{n=0}^{\infty} a_n (x-c)^n$$, divide the

$(n+1)$st term by the n^{th}.

Take absolute value of this quotient, then take the limit as $n \to \infty$. Call this limit L.

1) $L > 1$, series diverges
2) $L < 1$, series converges
3) $L = 1$, you know nothing.

Example 1: Consider $\sum_{n=0}^{\infty} \frac{x^n}{n!}$

Determine where this series converges, i.e., for what values of x.

Ratio test:

n^{th} term: $\frac{x^n}{n!}$

$(n+1)$st term: $\frac{x^{n+1}}{(n+1)!}$

Divide $\frac{(n+1)st}{n^{th}}$: $\frac{\left(\frac{x^{n+1}}{(n+1)!}\right)}{\left(\frac{x^n}{n!}\right)}$

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$$= \frac{x^{n+1}}{(n+1)!} \frac{n!}{x^n}$$ group like terms

$$= \frac{x^{n+1}}{x^n} \frac{n!}{(n+1)!}$$

$$= \cancel{\frac{x^n \cdot x}{x^n}} \frac{n!}{(n+1)!}$$

$$= x \frac{\cancel{n!}}{(n+1)\cancel{n!}} = \frac{x}{n+1}$$

Take absolute values

$$\frac{|x|}{n+1}$$

Take limit as $n \to \infty$

$$\lim_{n \to \infty} \frac{|x|}{n+1} = 0$$ no matter what x is

$0 < 1$, so by the ratio test,

$$\sum_{n=0}^{\infty} \frac{x^n}{n!}$$ converges for all values of x.