Announcements

1) Quiz Thursday Over 11.6+11.8

2) Exan next Thursday

Radius and Interval of Convergence Given a power series that is shifted $f(x) = \sum_{n=1}^{\infty} \alpha_n (x - c)^n$ $\Lambda = 0$ the series can coverge in one of three ways: () Only X=C 2) A finite interval centered at X=C 3) All real numbers So the series converges in an interval

called the interval of convergence

Italf the length of the interval of convergence is called the radius of convergence.

The ratio test almost always can find the radius of convergence, but can miss out a bit on the interval in case 2).

Example 1: Find the radius of convergence

of
$$\sum_{n=1}^{\infty} \frac{3^{n}(x-2)^{n}}{n+2}$$

nth term:
$$3^{n}(x-2)^{n}$$

nt2

1) Divide
$$(n+1)^{s+} = \frac{3^{n+1}(x-a)^{n+1}}{n+3} \frac{n+2}{3^n(x-a)^n}$$

a) Collect like terms:
$$\frac{3^{n+1}}{3^n} \frac{(x-2)^{n+1}}{(x-2)^n} \frac{n+2}{n+3}$$

3) Collect like terms:
$$\frac{3^{n+1}}{3^n} \frac{(x-2)^{n+1}}{(x-2)^n} \frac{n+2}{n+3}$$

$$= 3 (X-2) \underline{n+3}$$

31 Take absolute values 3 (x-2) <u>n+2</u> $= |3| |x-2| |\frac{n+2}{n+2}|$ $= 3 | x - 2 | \frac{n + 3}{n + 3}$ 4) Take limit as n-200 lin 3 (x-2) 1+2 1-20

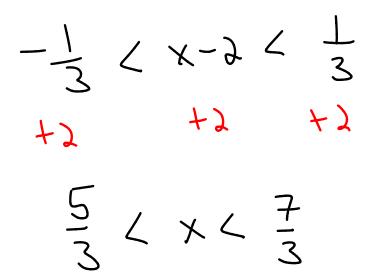
4) Take limit as
$$n \rightarrow \infty$$

$$\lim_{n \rightarrow \infty} 3 |x-2| \lim_{n \rightarrow \infty} \frac{n+2}{n+3}$$

$$= 3 |x-2| \lim_{n \rightarrow \infty} \frac{n+2}{n+3}$$

$$= 1$$

$$= 3 |x-2|$$
(5) Set less than one and solve
 $3 |x-2| < 1$
 $|x-2| < \frac{1}{3}$ means
 $-\frac{1}{3} < x-2 < \frac{1}{3}$



Ratio test says: - series converges when $\frac{5}{3} < x < \frac{7}{3}$ - series diverges when $x < \frac{5}{3}$ or $x > \frac{7}{3}$ - when $x = \frac{5}{3}$ or $x = \frac{7}{3}$, the test tells you nothing

Radius =
$$\frac{1}{2}$$
 (length of interval)
= $\frac{1}{2} \cdot (\frac{2}{3}) = \frac{1}{3}$

Example d: Find the radius of convergence
for
$$g(x) = \int_{n=1}^{\infty} (2n)! (x-6)^n$$

Definitely converges at x=6 (center). Ratio test n^{th} term: $(2n)!(x-6)^{n}$ $(n+1)^{st}$ term: $(2(n+1))!(x-6)^{n+1}$ $= (2n+2)!(x-6)^{n+1}$

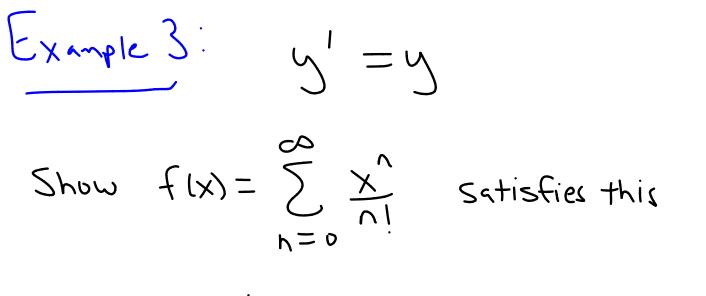
1) Divide
$$(6+1)^{st}$$
 term by nth term:
 $\frac{(2n+2)!(x-6)^{n+1}}{(2n)!(x-6)^n}$

2) Group like terms $\frac{(2n+2)!}{(2n)!}$ $\frac{(x-6)^{n+1}}{(x-6)^{n}}$ $= \frac{(2n+2)!}{(2n)!} (X-6)$ = (2n+2)(2n+1)(2n)!(x-6)Lan) = (2n+2)(2n+1)(X-6)

3) Take absolute values (2nt2) (2nt1) (x-6) = | 2nt2 | 2nt1 | 1x-6 | = (2nta) (2nti) [X-6] 4) Take limit as N>0 $\lim_{n \to \infty} (2ntd)(2ntl)|x-6| = \infty$ except when x=6. Then (2n+2)(2n+1)(6-6) $= (2n+2)(2n+1) \cdot 0 = 0$ so the limit is zero.

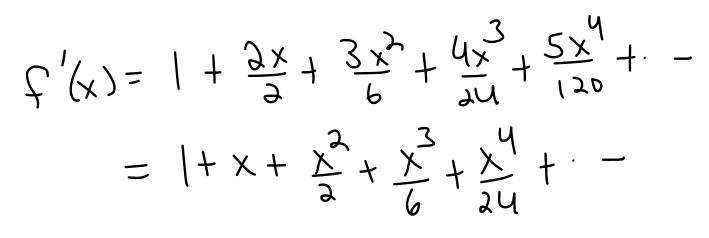
5) Set less than one But the limit is only less than one if $\chi = 6$. Then the interval of convergence is just x=6. Radius = 1 (length of interval) $=\frac{1}{\lambda}$ · 0 = 0 .

Power Series Calculus (Section 11.9) We would like to use power series to solve differential equations. To do that, we need to know how to take derivatives ! Given $f(x) = \sum_{n=1}^{\infty} a_n (x-c)^n$ with nonzero radius of convergence, $f(x) = q_0 + q_1(x-c) + q_2(x-c) + q_3(x-c) + \cdots$ $f'(x) = b + a_1 + a_2 \cdot a(x-c) + a_3 \cdot 3(x-c) + ...$



equation.

 $f(x) = [+x + \frac{x^{2}}{x^{2}} + \frac{x^{2}}{6} + \frac{x^{4}}{14} + \frac{x^{3}}{120} + \cdots$



= f(x)

This says $f(x) = Ce^{x}$ for some constant C. To find C, set x=0.

$$|=f(o)=Cc_{o}=C$$

$$S_{0} \quad f(x) = e^{x}$$