Announcements

1) Quiz l tomorrow over calc l material

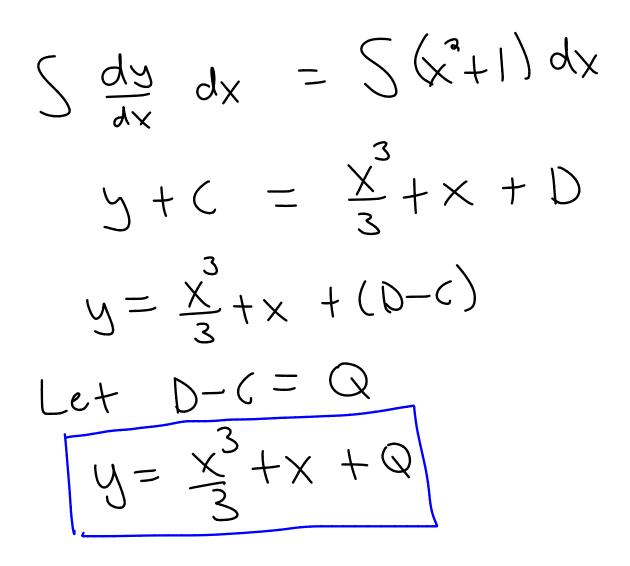
2) Review problems on (anvas under "Files"

What is calculus good for?

Solving differential equations What is a differential equation? An equation involving an Unknown function and its derivative (s)

Example 1: dy = x+1 ፈኦ

y = unknown function To get y, integrate both sides with respect to x:

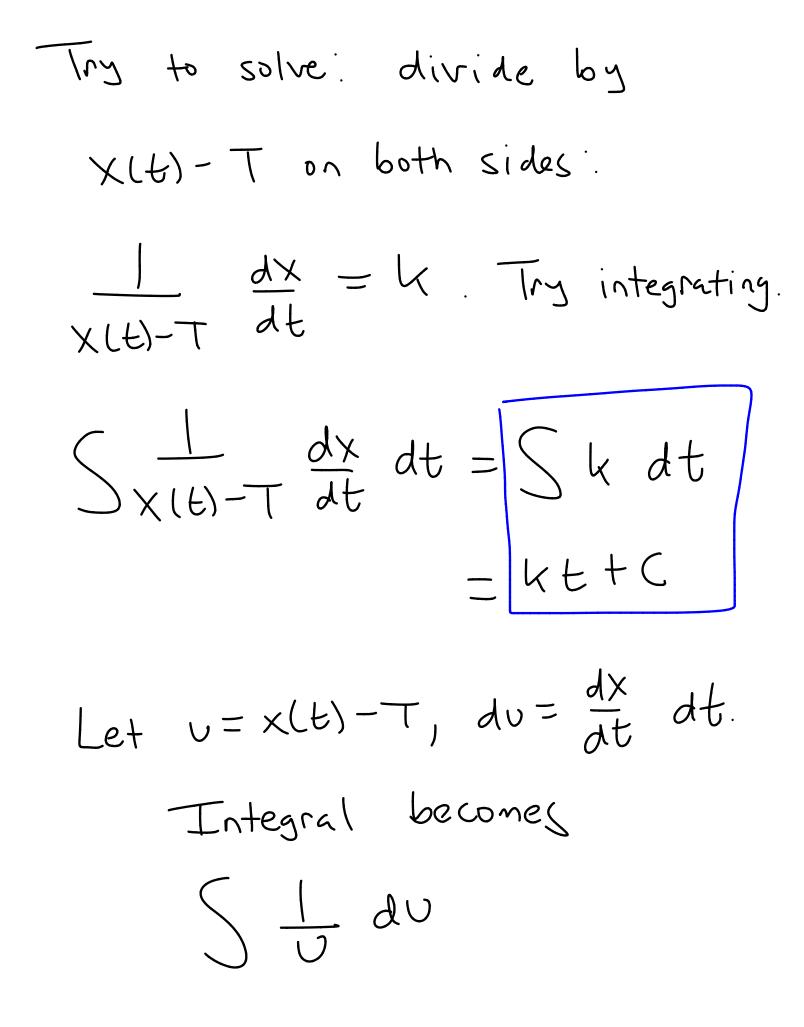


Moral: you only need one arbitrary constant Lin this (1955)

Example 2: (Newton's Law of Cooling)

Newton's Law of Cooling $\frac{dx}{dt} = k(x(t) - T)$ dt (an't directly integrate both sides Because we dont Know X(t), so we cant (alculate

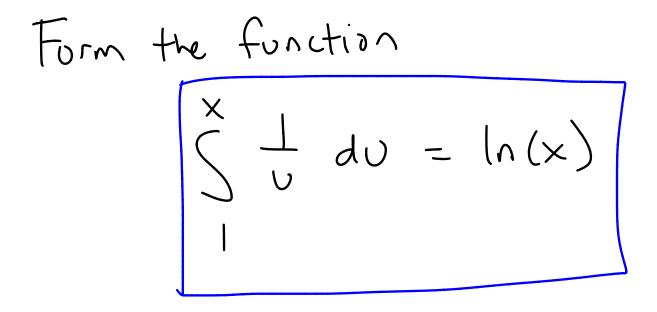
(x(t) dt)



What is Studu?

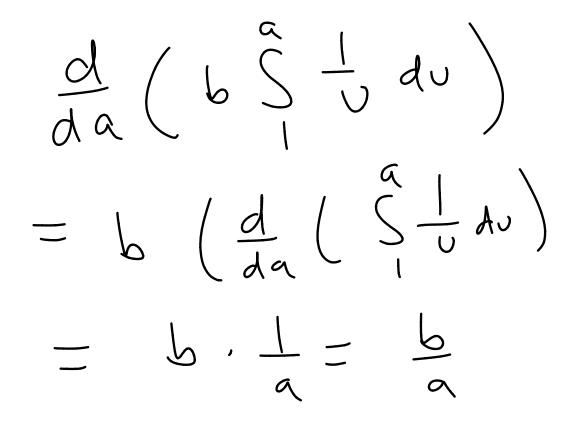
UZD

Concentrate on UZL.



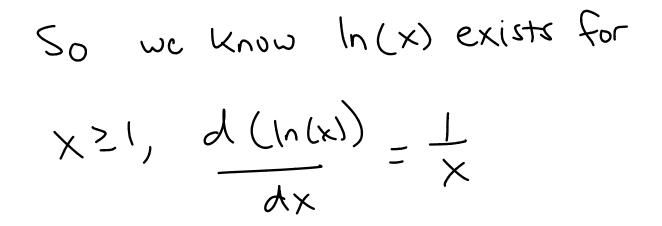
Log property: $ln(a^{b}) = b ln(a) (a21, b70)$ Verify this property for $\int_{-\infty}^{\infty} \frac{1}{\sqrt{2}} dv$: show $\int_{-1}^{q} \frac{1}{v} dv = b \int_{-1}^{q} \frac{1}{v} dv$ b = constant, a=variable. Let Differentiate both sides with respect to a.

By the Fundamental Theoren of Calculus, the righthand side is

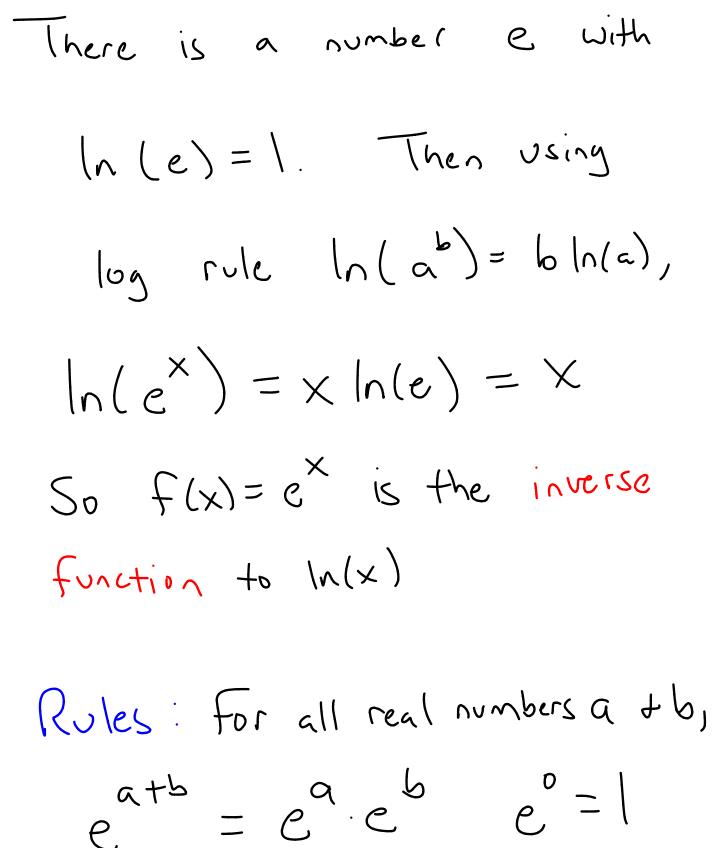


Again using Fundamental Theorem, $= \frac{1}{a^{b}} \cdot \frac{d}{da} \left(\frac{b}{a^{b}} \right)$ $= \frac{1}{a^{b}} \cdot \frac{b}{b} - \frac{b}{a^{b-1}}$ $\frac{1}{a} \cdot b \cdot a^{b} \cdot \frac{1}{a}$

This shows that two functions
have the same derivative,
so they differ by a constant
$$a^{b}$$
 $\frac{1}{2} dv - b \int \frac{1}{2} dv = C$
 $1 \qquad 1$
 $C = constant for allvalues of $a + b$.
Choose $a = 1$
 $\int \frac{1}{2} dv - b \int \frac{1}{2} dv = C$
 $\int \frac{1}{2} dv - b \int \frac{1}{2} dv = C$$



For $0 < x \leq 1$ $l_{n}(x) = - \int_{x}^{1} \frac{1}{v} dv$ $= \int_{x}^{x} \frac{1}{v} dv$ $= \int_{1}^{x} \frac{1}{v} dv$ $= \int_{1}^{x} \frac{1}{v} dv$



 $e^{ab} = (e^a)^b$

Derivative: X = In(eX) Differentiate both sides wrt X

and use the chain rule.

 $\frac{d}{dx}(x) = \frac{d}{dx}(\ln(e^{x}))$ $= \frac{1}{\rho^{\times}} \cdot \frac{d}{dx} \left(e^{\times} \right)$ multiply both sides by c?: $e^{X} = \frac{d}{dx} (e^{X})$

Example 3:

$$\left(-\frac{T}{2} \times \sqrt{\frac{T}{2}}\right) = \int \frac{\sin(x)}{\cos(x)} dx$$

$$v = cos(x)$$

 $dv = -sin(x) dx$

We get

$$-\int \int dv = -\ln(v) + C$$

$$= -\ln(\cos(x)) + C$$

Example 4:
$$\int_{0}^{2} \frac{x^{2}(10^{\sqrt{x^{3}+1}})}{\sqrt{x^{3}+1}} dx$$
Let $v = \sqrt{x^{3}+1} = (x^{3}+1)^{1/2}$
 $dv = \frac{1}{2} \cdot 3x^{2} \cdot (x^{3}+1)^{-1/2} dx$
 $= \frac{3}{2} \frac{x^{2}}{\sqrt{x^{3}+1}} dx$
 $\frac{2}{3} dv = \frac{x^{2}}{\sqrt{x^{3}+1}} dx$
 $v(0) = \sqrt{v^{3}+1} = 1$
 $v(1) = \sqrt{v^{3}+1} = 1$

The integral becomes $2/3 \int_{1}^{2} |0 dv|$ What is Slordu? Write IO = e In(IO) $= \rho$ Substitute one more time! Let $w = u \ln(10)$ $dw = \ln(10) dv, So$ $\frac{1}{\ln(10)} dw = dv$

$$\begin{split} & \omega(1) = \ln(10) \\ & \omega(3) = 3 \ln(10) = \ln(1000) \\ & \omega(3) = 3 \ln(10) = \ln(1000) \\ & \omega(3) = 3 \ln(10) \\ & \ln(1000) \\ & = \frac{2}{3} \ln(10) \\ & \ln(10) \\ & = \frac{2}{3} e^{\omega} \left[\frac{\ln(1000)}{\ln(10)} \\ & \ln(10) \\ & = \frac{2}{3} e^{\omega} \left[\frac{1980}{\ln(1000)} \right] \\ & = \frac{2}{3} e^{2} \frac{1980}{\ln(1000)} \\ & = \frac{10}{3} e^{2} \frac{10}{\ln(1000)} \\ & = \frac{10}{10} e^{2} \frac{10}{\ln(1000)} \\ & = \frac{10}$$

Example 5: (the only hard derivative)

 $f(x) = x_x$

Write $X = e^{\ln(X)}$ $= e^{\times |n(x)|}$

Differentiate

 $\frac{d}{dx} \begin{pmatrix} x^{\times} \end{pmatrix} = \frac{d}{dx} \begin{pmatrix} e^{x \ln(x)} \end{pmatrix}$ $= e^{\times \ln(x)} \frac{d}{dx} (\chi \ln(x))$ $= X \left(X \cdot \frac{1}{X} + \left[u(X) \right] \right)$ $= \left| \begin{array}{c} \times \\ \times \end{array} \right| \left(1 + \left(n(x) \right) \right)$