Announcements

1) HWY due Thursday

Jarametric Curves

(Section 10.1)

A way to unify rectangular and polar graphs by specifying and X function and a y function, both dependent on a single variable t Write: $\langle X(t), Y(t) \rangle$

Example 1: $f(t) = \langle cos(t), sin(t) \rangle$

$$X(t) = cos(t) (x - coordinate)$$

 $Y(t) = sin(t) (y - coordinate)$

$$X(t) = cos(t)$$
 (x-coordinate
 $Y(t) = sin(t)$ (y-coordinate
The graph is a circle, center
(0,0) and radius 1, since

$$x(t)^{2} + y(t)^{2}$$

$$= (os^{2}(t) + sin^{2}(t))$$

$$= ($$

How to graph: The graph of a parametric function is only points (XLt), y(t)) - you suppress the t value when graphing, so we get a 2-dimensional graph: a piece of string" in 2 dimensions

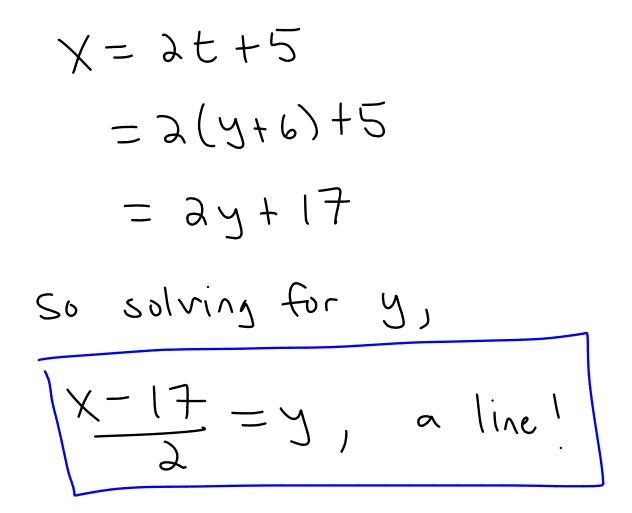
 $E_{\text{Xample } 2} = \langle 2t+5, t-6 \rangle$ Find a Cartesian equation $X(t) = \partial t + 5$ y(t) = t - 6Suppress & inputs $\chi = \lambda t + 5$

 $\dot{y} = t - b$

Solve for t in one equation. Try

y = t - 6, so t = y + 6.

Plug into equation for X



Example 3 : Parameterize y=x2.

There are infinitely many weys to do this, but perhaps the easiest is just to rename X = t, $y = x^2 = t^2$

50 (t,t^2) is a parameterization.

Parameterizing y=f(x)

We can generalize this last example to any function y=f(x). A parameterization is $\langle t, f(t) \rangle$

Example 4 : Parameterize $\Theta = \frac{\pi}{3}$

Calculus with Parametric Curves (Section 10.2) langent Lines Given a parametric curve $\langle \chi(t), \chi(t) \rangle$ Assume y can be expressed as a function of x (at least on small intervals of t).

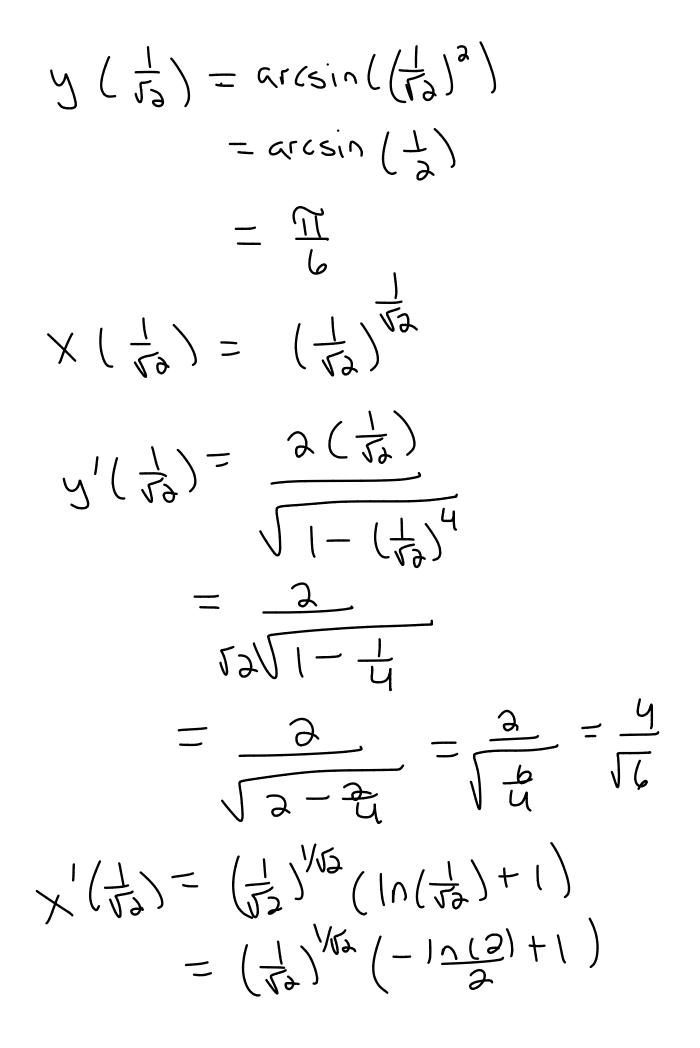
Using the Chain Rule, $\frac{dy}{dx} = \frac{dy}{dt} \frac{dt}{dx}$ $='' \begin{pmatrix} dy \\ dt \end{pmatrix}$ $\left(\begin{array}{c} dx\\ dx\end{array}\right)$

Slope of tangent line at t=a: ५'(१) $\frac{1}{\chi'(a)}$ Shorthand equation for tangent line: $\frac{y'(a)}{x'(a)} = \frac{y-y(a)}{x-x(a)}$

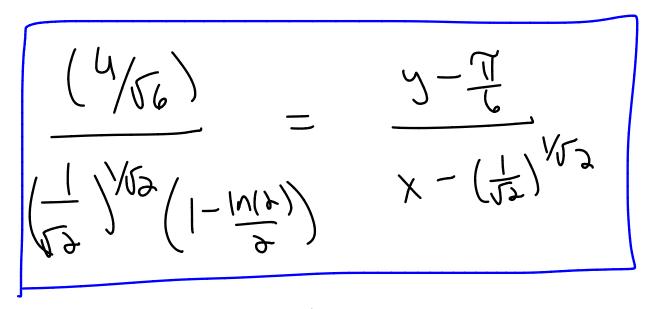
Example 1: Find the tangent line
to
$$f(t) = \langle t^{t}, \operatorname{arcsin}(t^{a}) \rangle$$

 $at t = \frac{1}{\sqrt{a}}$
Find $x'(t), y'(t)$.
 $y'(t) = at \frac{d}{dt} (\operatorname{arcsin})(t^{a})$
 $= at \frac{1}{\sqrt{1-(t^{a})^{a}}}$
 $= \frac{at}{\sqrt{1-t^{4}}}$

 $\chi(t) = t^{t} = e_{1} \ln(t^{t})$ = ethlt) $\chi'(t) = e^{t\ln(t)} \frac{d}{dt} (t\ln(t))$ $= t^{t} \left(\left| \cdot \left| h(t) + t \cdot \frac{1}{t} \right| \right)$ $= t^{+} (|n(t)+1)$ Equation for line: $y'(\frac{1}{6}) - y - y(\frac{1}{6})$ $\chi'(t_{a}) = \frac{1}{\chi - \chi(t_{a})}$



Final answer:



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