Announcements

1) Exam 2 pushed back

2) Mitchell's mentoring sessions:

Th 2-3 CB 1040

Kyce's mentoring sessions:

W 2-3 CB 2062

office hours

M Th 11-12

T 3:30-4:30

Polar Coordinates

(Section 10.3)

The Heat Equation governs the flow of heat through an object.

Unfortunately, the Heat Equation is usually over two or three dimensional regions and involves partial derivatives.

Polar Coordinates

A different way of describing points in 2 dimensions.

Given a point (x,y) in rectangular (cartesian) coordinates, the polar coordinates of (x,y) involve describing the point using a magnitude and a direction

Magnitude: $r = \sqrt{x^2+y^2}$

direction: $\theta = \arctan\left(\frac{y}{x}\right)$

(almost true ..)

From Polar to Rectangular

Given a point (r, θ),

the rectangular coordinates are

$$\boxed{x = r\cos\theta, \quad y = r\sin\theta}$$

Always true

Example 1: Convert $(2, -\pi/2)$ to rectangular coordinates

$$x = r\cos\theta = 2\cos(-\pi/2) = 0$$

$$y = r\sin\theta = 2\sin(-\pi/2) = -2$$

$$\boxed{(0, -2)}$$

Example 2: Convert $(-\sqrt{3}, 1)$ to polar coordinates

$$r = \sqrt{x^2+y^2} = \sqrt{3+1} = 2$$

$$\theta = \arctan\left(\frac{y}{x}\right) = \arctan\left(\frac{1}{-\sqrt{3}}\right)$$

$=$ some number in $(-\pi/2, \pi/2)$

$$= -\frac{\pi}{6}$$

$\left(2, -\frac{\pi}{6}\right)$ in 4^{th} quadrant

$(-\sqrt{3}, 1)$ is in 2^{nd} quadrant, so this can't be right!

The Complete Rules

If (x,y) is in the first or fourth quadrant, the polar coordinates are

$$r = \sqrt{x^2+y^2}, \quad \theta = \arctan\left(\frac{y}{x}\right)$$

$$(x \neq 0)$$

If (x,y) is in the second or third quadrant, the polar coordinates are

$$r = \sqrt{x^2+y^2}, \quad \theta = \arctan\left(\frac{y}{x}\right) + \pi$$

$$(x \neq 0)$$

Finishing Example 2:

$$r = 2$$

$$\theta = \arctan\left(\frac{y}{x}\right) + \pi$$

$$= -\frac{\pi}{6} + \pi$$

$$= \frac{5\pi}{6}$$ in 2nd quadrant ✓

If $x=0$, the polar coordinates

are $\left(y, \frac{\pi}{2}\right)$ if $y>0$

$\left(-y, -\frac{\pi}{2}\right)$ if $y<0$

For example, the polar coordinates

of $(0,-3)$ are

$\left(3, -\frac{\pi}{2}\right)$

Graphing in Polar vs. Rectangular

A polar function is of the form $r = f(\theta)$.

To convert from polar to cartesian and vice-versa, use some of the identities:

$$r = \sqrt{x^2 + y^2}$$

$$\theta = \arctan\left(\frac{y}{x}\right)$$

$$x = r\cos\theta$$

$$y = r\sin\theta$$

Example 3: $x^2+y^2=23$

$$\underbrace{\sqrt{x^2+y^2}} = \sqrt{23}$$

$$\boxed{r=\sqrt{23}}$$

for polar coordinates

Example 4: $r = 2\sin(\theta)\cos(\theta)$

Convert to rectangular:

$$r = \sqrt{x^2+y^2}$$

$x = r\cos\theta$, so

$$\cos\theta = \frac{x}{r} = \frac{x}{\sqrt{x^2+y^2}}$$

Similarly,

$$\sin\theta = \frac{y}{\sqrt{x^2+y^2}},$$ so

substituting:

$$\sqrt{x^2+y^2} = \frac{2xy}{x^2+y^2}$$

$$(x^2+y^2)^{3/2} = 2xy,$$

$$\boxed{(x^2+y^2)^{3/2} - 2xy = 0}$$