Math 215 Homework 1

READ ME: Except where indicated, merely finding the answer to a problem is not enough to receive credit. You must show how you arrived at that answer.

1) Find an equation for the plane that passes through the point $(2, 0, 1)$ and is parallel to $4x + 2y - 7z + 5 = 0$.

2) A plane P contains the points $(-4, 0, 9)$, $(6, -10, -5)$, and $(7, -2, 3)$.

a) Find a UNIT normal vector to P.

b) Use your answer from part a) to determine the equation of P.

3) Calculate the area of the parallelogram determined by the vectors $\langle 6, -20, 8\rangle$ and $\langle -13, 1, -9\rangle$.

4) You are using your favorite wrench, Trusty, to turn a bolt. If Trusty is 30 cm long and you apply a force of 2000N (roughly 1/4 the force of your average alligator bite) at an angle of 30 degrees ($\pi/6$ radians) from horizontal, how much torque does the unfortunate bolt endure?

5) Determine the equation of the line orthogonal to the plane $3x-8y+17z = -34$ that passes through the point $(-1, 5, -21)$.

6) Find a parametric equation for the line

$$l_1 := x + 1 = \frac{2 - y}{3} = \frac{z - 1}{2}.$$

Then determine whether l_1 intersects the line

$$l_2 := \frac{1 - x}{2} = \frac{y + 1}{6} = \frac{-z}{3}.$$

If they intersect, find the point of intersection.

7) Compute the derivative of $f(t) = \left\langle \arctan(t^2 + 1), e^{\sin(t)}, \ln\left(\frac{2t + 1}{t - 6}\right)\right\rangle$.

8) Find the equation of the tangent line to the graph of $g(t) = \langle \cos(\pi t^3), t\ln(t)\rangle$ at the point $(-1, 0)$.

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9) Determine whether $\lim_{t\to\infty} f(t)$ exists if

$$f(t) = \left\langle \sqrt{t+2} - \sqrt{t}, \left(\frac{t}{3+t}\right)^t \right\rangle.$$

If the limit exists, compute it.

10) (4 points) Describe the surface that the curve $\langle t\sin(t), t, t\cos(t)\rangle$ lies on both geometrically and with equations (no justification necessary). Then use this to sketch the curve, complete with arrows indicating the direction of increasing time.

11) If f is a vector-valued function with values in two dimensions and its magnitude is a nonzero constant, what geometric shape does the graph of f live on? Without actually doing any calculations, draw a picture to find the dot product of $f(a)$ with its derivative at $t = a$.

12) Establish a general formula for the derivative of $\|f(t)\|^2$ where f is vector-valued in three dimensions and use this to get the analogous result from problem **5)** in three dimensions.

13) If $f(t) = \langle -t^3, 2t, 4t^2\rangle$, find the derivative of $f' \times f''$ using the formula $\frac{d}{dt}(g \times f) = g \times f\,' + g\,' \times f$.

14) Let v_1, v_2, and v_3 be nonzero, pairwise orthogonal vectors in three dimensions. Show that if there are real numbers a_1, a_2, and a_3 with

$$a_1v_1 + a_2v_2 + a_3v_3 = 0$$

then $a_1 = a_2 = a_3 = 0$.