

Math 215 Homework 3- Continued

READ ME: Except where indicated, merely finding the answer to a problem is not enough to receive credit. You must show how you arrived at that answer.

1) Without using the definition, show that

$$\lim_{(x,y) \rightarrow (1,2)} \sin((x-1)^4) \arctan\left(\frac{1}{x^2 + y^2 - 2(x+2y) + 5}\right) = 0.$$

2) Let

$$f(x, y) = \begin{cases} \frac{x^3y - xy^3}{x^2 + y^2} & (x, y) \neq (0, 0) \\ 0 & (x, y) = (0, 0) \end{cases}$$

a) Find $\frac{\partial f}{\partial x}$ and $\frac{\partial f}{\partial y}$ when $(x, y) \neq (0, 0)$.

b) Use the definition to find $\frac{\partial f}{\partial x}$ and $\frac{\partial f}{\partial y}$ at $(0, 0)$.

c) Show that the values of $\frac{\partial^2 f}{\partial x \partial y}$ and $\frac{\partial^2 f}{\partial y \partial x}$ are not equal at $(0, 0)$. Does this contradict Clairaut's theorem?

3) Show that if f and g are differentiable real-valued functions of a single variable, then $z = f(x + 6t) + g(x - 6t)$ is a solution of the equation

$$\frac{\partial^2 z}{\partial t^2} = 36 \frac{\partial^2 z}{\partial x^2}.$$