Name:

Math 215 Practice Final

Directions: Except where indicated, show all work. Answers without justification are worth close to nothing. DO NOT convert roots or transcendentals into decimal approximations; just leave them as they are.

1) Give an equation for the tangent plane and normal line to the surface $5 = \sqrt{x^2 + y}$ at the point $(6, -11, 1013)$.

2) a) Find the direction of maximum increase of the function $f(x, y, z) = \dfrac{xyz}{x + yz}$ at the point $(-5, 5, 2)$.

b) Calculate the directional derivative of the function in part a) in the direction of the vector $\langle 6, 8, 0 \rangle$

3) Locate and classify all critical points (i.e. are they local maxima, minima, saddle points, or neither) for the function

$$f(x, y) = 6x^3 - 9y^2 + 18xy.$$

4) a) Find the vector projection of $\langle 0, 3, -3\rangle$ onto $\langle 6, -3, 6\rangle$. The formula for the projection of v onto w is

$$\frac{v \cdot w}{\|w\|} w$$

b) If a vector v forms an angle of $\frac{\pi}{3}$ with $w = \langle 4, 3, -7\rangle$ and the projection of v onto w has length equal to 8, find the length of v. NOTE: **you do not actually have to find** v.

5) Let f be a differentiable function of one variable and let

$$g(x, y) = f(\ln(x^2 - y^2)).$$

Verify that

$$y\frac{\partial g}{\partial x} + x\frac{\partial g}{\partial y} = 0.$$

6) Consider the integral $\displaystyle\int_0^1 \int_{e^y-1}^{e-1} \sqrt{(x+1)\ln(x+1) - x}\; dx\; dy$.

a) Draw the region of integration.

b) Evaluate the integral.

7) Set up **but do not solve** an integral in SPHERICAL coordinates for the mass of the solid given by the region in front (i.e. the direction of the positive y-axis) of the plane $x = \sqrt{3}y$ and inside the sphere $x^2 + y^2 + (z-2)^2 = 4$ if the density function is given by the distance from a point in the solid to the xz-plane.

8) a) Is the vector field $\langle \sqrt{x^3y^5}+y, \sqrt{x^5y^3}+x \rangle$ conservative or not? (I won't test you over this, but it is good practice in taking partials!)

9) Show that

$$\lim_{(x,y)\to(2,0)} \frac{8xy^4 - 16y^4}{\tan((x-2)^2) + y^8}$$

does not exist.