Name:

Math 215 Exam 1

October 4, 2012

Directions: WRITE YOUR NAME ON THIS EXAM! Except where indicated, merely finding the answer to a problem is not enough to receive full credit; you must show how you arrived at that answer. DO NOT convert irrational numbers such as $\sqrt{3}$ or π into decimal approximations; just leave them as they are.

1) Given the vectors $\vec{v} = \langle -1, 6, 8 \rangle$ and $\vec{w} = \langle -5, 3, 10 \rangle$, find

    a) (2 points) $\|\vec{v}\|$ (Answer: $\sqrt{101}$)

    b) (6 points) $\vec{v} \cdot \vec{w}$ (Answer: 103)

    c) (8 points) the angle θ between $\vec{v}$ and $\vec{w}$, correct to the nearest degree. (Answer: 28 degrees)

2) PICK ONE: circle your choice. If you do both, I will grade the one you do WORSE on.

a) (20 points) Boris Johnson, Mayor of London, is zip lining from a height of 148 feet. Boris becomes stuck at 33 feet above the ground with 65 feet of wire remaining. If Boris weighs 190 pounds, the wire is 1050 feet long and the terminal end is 45 feet above the ground, find the tension at either end of the wire, correct to the nearest foot-pound. (Answer: 626, 633)

-OR-

b) (20 points) Let $f(t) = \langle \sin(\pi t), \cos(\pi t), \ln(t) \rangle$. Find $T(t)$, $N(t)$, and $B(t)$ at the point $\langle 0, -1, 0 \rangle$.

3) (15 points) Determine the equation of the plane containing the lines

$$f(t) = \langle -21, 16, 8 \rangle + t\langle 2, 0, 3 \rangle$$

and

$$g(t) = \langle 19, -3, 51 \rangle + t\langle -40, 19, -43 \rangle.$$

(Answer: $\langle 840, 1223, -241 \rangle \cdot \langle x + 21, y - 16, z - 8 \rangle = 0$)

4) (16 points) Find the arc length of the portion of the curve

$$f(t) = \langle t - \tan(t), \sqrt{2}\sec(t), \tan(t)\rangle$$

from $t = 0$ to $t = \dfrac{\pi}{4}$. (Answer: $2 - \frac{\pi}{4}$)

5) (23 points) Determine

$$\lim_{t \to 0^+} \left\langle \frac{e^{2t} - 1}{t}, \csc(t) - \frac{1}{t}, t^{\sqrt{t}} \right\rangle.$$

You may assume $0 < t < \pi$. (Answer: $\langle 2, 0, 1 \rangle$)