Name:

Math 215 Exam 2

November 5th, 2015

Directions: WRITE YOUR NAME ON THIS EXAM! Except where indicated, merely finding the answer to a problem is not enough to receive full credit; you must show how you arrived at that answer. IF you convert irrational numbers such as $\sqrt{3}$ or π into decimal approximations, round to at least 4 decimal points.

1) (10 points, 2 points each) True/False. No justification is necessary.

a) For all functions $z = f(x, y)$, $\dfrac{\partial^2 f}{\partial x \partial y} = \dfrac{\partial^2 f}{\partial y \partial x}$ wherever the partials exist.

b) If $z = f(x, y)$ and $x = x(s, t)$, $y = y(s, t)$, and $g(s, t) = f(x(s, t), y(s, t))$, then $\dfrac{\partial g}{\partial s} = \dfrac{\partial f}{\partial x}\dfrac{\partial x}{\partial s}$.

c) The gradient vector for a function $f(x, y)$ at (x_0, y_0) is orthogonal to the level curve $k = f(x, y)$ where $k = f(x_0, y_0)$.

d) The curvature of a circle increases as the radius of the circle increases.

e) If a function $z = f(x, y)$ is continuous at the point (a, b), then

$$\lim_{(x,y)\to(a,b)} f(x, y) = f(a, b).$$

2) Let

$$g(x, y) = \frac{xy - y^2}{y - x}.$$

a) (6 points) Compute $\dfrac{\partial^2 g}{\partial x \partial y}$.

b) (5 points) Find the directional derivative of g at the point $(1, 2)$ in the direction of the vector $\langle 3, 4 \rangle$.

c) (4 points) Find the direction and the magnitude of maximum increase of g at the point $(1, 2)$.

3) (20 points) Find the equation of the tangent plane to the level surface

$$2 = \arctan(xy + yz - xz) + z$$

at the point $(-1, -2, 2)$.

4) (25 points) Compute the curvature of the parametric function

$$f(t) = \langle \cos(3\pi t), \sin(2\pi t), t^t \rangle$$

at the point $t = 1$. *Hint:* use the cross-product formula and plug in $t = 1$ before you take the cross-product.

5) (20 points) Show that

$$\lim_{(x,y)\to(3,-2)} \frac{xy^2 + 4xy + 4x - 3y^2 - 12y - 12}{2(x-3)^2 + (y+2)^4}$$

does not exist.