Name:

Math 215 Practice Exam 2

March 18th, 2010

1) a) (7 points) If $f(x, y) = \ln\left(\dfrac{2x}{y}\right)$, find the direction of maximum decrease of f at the point $(3, -1)$ (answer: $\langle -1/3, -1 \rangle$)

b) (3 points) Calculate the magnitude of the rate of change in the direction of maximum decrease. (answer: $\sqrt{10}/3$

c) (8 points) Compute the directional derivative of the function in part a) at the point $(3, -1)$ in the direction of the vector $\langle 42, 56 \rangle$. Be sure to simplify your answer. (answer: 1)

2) (15 points) Find the equation of the tangent plane to the graph of $z = \arctan(x^2 y)$ when $x = 1$ and $y = \sqrt{3}$. (answer: $\langle \sqrt{3}/2, 1/4, -1 \rangle \cdot \langle x - 1, y - \sqrt{3}, z - \pi/3 \rangle = 0$)

3) (15 points) Find and classify (i.e. are they local maxima, minima, or saddle points) all critical points for the function $f(x, y) = 2x^2 - 2xy + y^2 - 4y$. (answer: $(2,4)$ is the only critical point and is a local minimum.)

4) (12 points) Determine the maximum and minimum values of the function $f(x, y) = 2x^2 - 2xy + y^2 - 4y$ (note that this is the same function from problem #4) over the closed region bounded by the square with vertices $(0, 0)$, $(0, 6)$, $(6, 0)$, and $(6, 6)$. (answer: the max is 72, the min is -8)

5) (15 points) Show that

$$\lim_{(x,y)\to(1,3)} \frac{y\sin(x-1) - 3\sin(x-1)}{(x-1)^2 + (y-3)^2}$$

does not exist (answer: try the curves $y = 3$ and $x - 1 = y - 3$).