Math 215 Exam 3

December 6, 2012

Directions: WRITE YOUR NAME ON THIS EXAM! Except where indicated, merely finding the answer to a problem is not enough to receive full credit; you must show how you arrived at that answer. DO NOT convert irrational numbers such as $\sqrt{3}$ or π into decimal approximations; just leave them as they are.

1) Let $\vec{F}(x, y, z) = \langle \sec(xy), x^2 - zy, e^{xz} \rangle$.

    a) (6 points) Calculate the divergence of $\vec{F}$.

    b) (8 points) Calculate the curl of $\vec{F}$.

    c) (6 points) Show $div(curl(\vec{F})) = 0$.

2) Let E be the region in $\mathbb{R}^3$ bounded by the cylinders $x^2+y^2 = 16$, $x^2+y^2 = 25$, the xy-plane, and the plane $z = 7$.

    a) (5 points) Draw the region E, labeling your picture carefully.

    b) (20 points) Determine $\displaystyle\int_E \sin\left(\sqrt{x^2+y^2}\right) \, dV$.

3) Let R be the region in the first quadrant enclosed by the curves $y = x$ and $x = y^3$.

a) (5 points) Draw R, labeling your picture carefully.

b) (20 points) Let C be the boundary of R, oriented counterclockwise. Evaluate the line integral $\int_C (y+e^{x^2})\, dx + (9x - \arctan(\sqrt{y}))\, dy$. (*Hint:* what is the color of money?)

4) (20 points) Compute the line integral $\int_C e^{xy}\,dx + \left(\dfrac{xye^{xy} - e^{xy}}{y^2}\right)dy$ where C is the curve $\langle t^{t^t}, \cos(\pi t/6)\rangle$ from $t = 1$ to $t = 2$.