Name:

Math 215 Exam 3

December 3, 2015

Directions: WRITE YOUR NAME ON THIS EXAM! Except where indicated, merely finding the answer to a problem is not enough to receive full credit; you must show how you arrived at that answer. IF you convert irrational numbers such as $\sqrt{3}$ or π into decimal approximations, round to at least 4 decimal points.

1) (10 points, 2 points each) True or false. No justification necessary.

a) The ellipse $\dfrac{x^2}{4} + \dfrac{y^2}{9} = 1$ bounds a region in $\mathbb{R}^2$ that is both Type I and Type II.

b) $\displaystyle\int_{\mathcal{R}} \sin(x)\cos(y)\ dA = \int_a^b \sin(x)\ dx \cdot \int_c^d \cos(y)\ dy$ if $\mathcal{R} = [a,b] \times [c,d]$.

c) A continuous, real valued function on a closed region $\mathcal{R}$ in $\mathbb{R}^2$ attains its maximum and minimum on $\mathcal{R}$.

d) If (a,b) is a critical point of $z = f(x,y)$ and f is differentiable, then

$$\frac{\partial f}{\partial x}(a,b) = \frac{\partial f}{\partial y}(a,b) = 0.$$

e) In polar coordinates, $r = x^2 + y^2$.

2) (21 points) Find and classify (i.e. are they local maxima, minima, or saddle points) all critical points for the function $f(x, y) = x^2y + 2y^2 + x^2$.

3) (18 points) Find the closest point(s) to the origin on the surface

$$x^2 + y + z = 5.$$

4) Let R be the region in the first quadrant enclosed by the curves $x^2+y^2 = 4$ and $x^2 + y^2 = 9$.

a) (5 points) Draw R, labeling your picture carefully.

b) (19 points) Determine $\int_R 10^{x^2+y^2}\ dA$.

5) Let R be the region in the first quadrant enclosed by the y-axis and the curves $y = 1$ and $y = x$.

a) (5 points) Draw the region R, labeling your picture carefully.

b) (22 points) Determine $\displaystyle\int_R x^2 \sin(\pi y^2)\, dA$.