Name:

# Math 215 Practice Exam 3

**1)** (15 points) Find inequalities in SPHERICAL coordinates for the region above the plane $z = 5$ and inside the sphere $z^2 + x^2 + y^2 - 10z = 0$.

**2)** Let $\vec{F}(x, y, z) = \langle \arctan(xy), \ln(x^2 + (y+1)^2), x \rangle$.

a) Determine whether $\vec{F}$ is conservative.

b) Calculate $div(\vec{F})$ and $curl(\vec{F})$.

c) Demonstrate that $div(curl(\vec{F})) = 0$.

**3)** If $C$ is the path (oriented counterclockwise) determined by the triangle with vertices $(0, 0)$, $(1, 0)$, and $(1, 2)$, calculate

$$\int_C xy \, dx + x^2 \, dy.$$

**4)** Consider the integral $\int_{\mathcal{R}} (x^4 - y^4)e^{2xy} \, dx \, dy$ where $\mathcal{R}$ is the region in the first quadrant bounded by the $y$-axis, $x^2 + y^2 = \sqrt{2}$, $x^2 - y^2 = 1/4$, and $x^2 - y^2 = 4/9$.

a) (5 points) Sketch the region $\mathcal{R}$

b) (8 points) Find the Jacobian of the transformation

$$T(u, v) = \left( \frac{\sqrt{u+v}}{\sqrt{2}}, \frac{\sqrt{u-v}}{\sqrt{2}} \right)$$

c) (12 points) Evaluate the integral, using any method at your disposal.