Name:

Math 215 Final

December 14, 2012

Directions: WRITE YOUR NAME ON THIS EXAM! Except where indicated, merely finding the answer to a problem is not enough to receive full credit; you must show how you arrived at that answer. DO NOT convert irrational numbers such as $\sqrt{3}$ or π into decimal approximations; just leave them as they are.

Now, if you had a choice on whether to take this exam, you must also sign to indicate your understanding of the following statement:

I understand that in choosing to take this final, I may lower my grade from what it was before the final.

SIGNED:________________________________

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1) Given the vectors $\vec{v} = \langle 2, -8, 0 \rangle$ and $\vec{w} = \langle -9, 1, -3 \rangle$, calculate

a) (2 points) $\|\vec{v}\|$

b) (4 points) $\vec{v} \cdot \vec{w}$

c) (6 points) the angle θ between $\vec{v}$ and $\vec{w}$, correct to the nearest degree.

2) Let $\vec{F}(x, y, z) = \langle 2x^3 - y, \ln(x^4 z), \tan(yz) \rangle$.

a) (4 points) Calculate the divergence of $\vec{F}$.

b) (8 points) Calculate the curl of $\vec{F}$.

c) (6 points) Show $div(curl(\vec{F})) = 0$.

3) (12 points) Find the equation of the tangent line to $\vec{r}(t) = \langle \cos(\pi t), e^{3t}, \sqrt{t+1} \rangle$ at the point $t = 0$.

4) Let $f(x,y) = \sin(x^2 - y)$.

a) (6 points) Find the direction of maximum increase of f at the point $(0, \pi/3)$.

b) (3 points) Calculate the magnitude of the rate of change of f in the direction of maximum increase at the point $(0, \pi/3)$.

c) (8 points) Compute the directional derivative at the point $(0, \pi/3)$ in the direction of the vector $\langle 6, -84 \rangle$. Be sure to simplify your answer.

5) (12 points) Determine the equation of the tangent plane to the graph of $f(x, y) = \arctan(x^4 - y^3)$ at the point $(1, 0)$.

6) (15 points) Find three positive numbers that sum to 15 such that the sum of their square roots is as small as possible.

7) a) (10 points) Set up an integral representing the arc length of the curve $r(t) = \langle 3\cos(2t), 3\sin(2t), \sqrt{13}t \rangle$ from $t = 12$ to $t = 18$.

b) (6 points) Compute the arc length.

8) Consider the integral $\int_{\mathcal{R}} \ln(x^2 + y^2)\, dA$ where $\mathcal{R}$ is the region in the first quadrant of $\mathbb{R}^2$ inside the circle $x^2 + y^2 = 36$ and below the line $y = x$.

a) (5 points) Sketch the region of integration $\mathcal{R}$.

b) (15 points) Evaluate the integral.

9) Consider the integral $\displaystyle\int_{\mathcal{E}} \cos((x^2+y^2+z^2)^{3/2})\, dV$ where $\mathcal{E}$ is the region inside (or below, if you prefer) the sphere $x^2+y^2+z^2=36$ and inside (or above, if you prefer) the cone $z=\sqrt{x^2+y^2}$ in $\mathbb{R}^3$.

a) (5 points) Sketch the region of integration $\mathcal{E}$.

b) (15 points) Evaluate the integral.

10) (11 points) Show that

$$\lim_{(x,y)\to(-2,0)} \frac{xy+2y}{(x+2)^2+y^2}$$

does not exist.

11) (12 points) Compute the line integral $\displaystyle\int_C e^{xy}\ dx + \left(\frac{xye^{xy} - e^{xy}}{y^2}\right)\ dy$ where C is the curve $\langle t^{t^t}, \cos(\pi t/6)\rangle$ from $t = 1$ to $t = 2$.

12) (15 points) Let $\vec{F}(x, y, z) = \langle 6x, -2y, z \rangle$. Evaluate the line integral

$$\int_C \vec{F} \cdot d\vec{r}$$

where C is the triangle in the first octant bounding the plane $z = 5 - x - 3y$.
Hint: use Stokes' Theorem.