Name:

Math 215 Final

December 15, 2015

Directions: WRITE YOUR NAME ON THIS EXAM! Except where indicated, merely finding the answer to a problem is not enough to receive full credit; you must show how you arrived at that answer. IF you convert irrational numbers such as $\sqrt{3}$ or π into decimal approximations, round to at least 4 decimal points.

Now, if you had a choice on whether to take this exam, you must also sign to indicate your understanding of the following statement:

I understand that in choosing to take this final, I may lower my grade from what it was before the final.

SIGNED:________________________________

1) Given the vectors $v = \langle 4, -1, 3 \rangle$ and $w = \langle -2, -5, 0 \rangle$, calculate

a) (4 points) $\|v + w\|$

b) (6 points) $v \times w$

c) (4 points) the angle θ between v and $v \times w$, correct to the nearest degree.

2) (10 points) Find the equation of the plane parallel to the plane

$$-5x + 17y - 9z = 2$$

and containing the point $(-1, 1, 15)$.

3) (12 points) Find the equation of the tangent line to

$$r(t) = \left\langle \tan(t), \sin(2t), t^2 - \frac{\pi^2}{16} \right\rangle$$

at the point $(1, 1, 0)$.

4) Let

$$g(x,y) = \frac{2xy^2 - 2x^3y}{x^2 - y}.$$

a) (6 points) Compute $\dfrac{\partial^2 g}{\partial y \partial x}$.

b) (5 points) Find the directional derivative of g at the point $(3, -4)$ in the direction of the vector $\langle 8, 15 \rangle$.

c) (4 points) Find the direction and the magnitude of maximum increase of g at the point $(3, -4)$.

5) (12 points) Determine the equation of the tangent plane to the graph of

$$3 = \cos(\pi xy) + xy + e^z$$

at the point $(1, -1, \ln(5))$.

6) (15 points) Find three positive numbers whose product is 125 and whose sum is a minimum.

7) a) (10 points) Set up an integral representing the arc length of the curve

$$r(t) = \left\langle t, t^2, \frac{4t^{3/2}}{3} \right\rangle$$

from $t = 1/2$ to $t = 5/2$.

b) (6 points) Compute the arc length.

8) Let $\mathcal{R}$ be the region in the fourth quadrant of $\mathbb{R}^2$ that is bounded by the x-axis, the circle $x^2 + y^2 = 64$, and the line $y = -x$.

a) (5 points) Draw R, labeling your picture carefully.

b) (15 points) Evaluate the integral $\int_{\mathcal{R}} (x^2 + y^2)^{1/3} \, dA$.

9) Let $\mathcal{R}$ be the region in the first quadrant of $\mathbb{R}^2$ that is bounded by the curve $x = y^{1/5}$, the line $x = 1$, and the x-axis.

a) (5 points) Draw R, labeling your picture carefully.

b) (15 points) Evaluate the integral $\displaystyle\int_{\mathcal{R}} \frac{1}{1+x^6}\, dA$.

10) (11 points) Show that

$$\lim_{(x,y)\to(0,0)} \frac{xy}{4x^2 + 9y^2}$$

does not exist.

11) (15 points) Find the integral of $f(x, y, z) = x$ over the tetrahedron with vertices $(0, 0, 0)$, $(0, 1, 0)$, $(3, 0, 0)$, and $(0, 0, 1)$.

12) Let $\mathcal{E}$ be the region in $\mathbb{R}^3$ that is inside the sphere $x^2 + y^2 + z^2 = 36$, outside the sphere $x^2 + y^2 + z^2 = 4$, and above the xy-plane.

a) (5 points) Sketch the region of integration $\mathcal{E}$.

b) (15 points) Evaluate the integral $\displaystyle\int_{\mathcal{E}} \arctan\left(\sqrt{\frac{x^2 + y^2}{z^2}}\right)\, dV$