Name:

Math 215 Practice Final

1) Given the vectors $v = \langle 1, 4, 7 \rangle$ and $w = \langle 8, -2, 6 \rangle$, calcuate

a) (4 points) $v \cdot w$

b) (6 points) $v \times w$

c) (2 points) $v \cdot (v \times w)$

2) (12 points) Find the equation of the tangent line to the curve $r(t) = \langle t, \cos(\pi t), \ln(t)\rangle$ when $t = 1$

3) Let $f(x, y) = \arctan(4x + y)$.

a) (6 points) Find the direction of maximum increase of f at the point $(-1, 4)$.

b) (3 points) Calculate the magnitude of the rate of change in the direction of maximum increase.

c) (8 points) Compute the directional derivative at the point $(-1, 4)$ in the direction of the vector $\langle 9, -40 \rangle$. Be sure to simplify your answer.

4) (12 points) Determine the equation of the tangent plane to the graph of the surface $x^2 + y^3 + z^4 = 18$ at the point $(1, 1, 2)$.

5) (20 points) Locate and classify all critical points (i.e. are they local maxima, minima, or saddle points) of the function $f(x, y) = x^3 + y^3 + 9xy + 1$.

6) a) (10 points) Set up an integral representing the arc length of the curve $r(t) = \langle \sin(t) - t\cos(t), \cos(t) + t\sin(t), t^2/2 \rangle$ from $t = 0$ to $t = 5$.

b) (8 points) Compute the arc length.

7) (15 points) Find $\dfrac{\partial z}{\partial x}$ at $(3,0)$ if $x^2z^5 + x^3y^4 - 8 = 2^{zy^2}$.

8) Given the integral $\displaystyle\int_0^8 \int_{\sqrt[3]{y}}^2 \sin(\pi x^2)\; dx\; dy$,

a) (5 points) Sketch the region of integration.

b) (15 points) Evaluate the integral.

9) (15 points) Find inequalities in SPHERICAL coordinates for the region above the xy-plane, below (or outside, if you prefer) the cone $z^2 = x^2 + y^2$ and inside the cylinder $x^2 + y^2 = 9$.

10) An object occupies the region inside the ellipsoid $\dfrac{x^2}{4} + y^2 + \dfrac{z^2}{25} = 1$ and above the xy-plane.

a) (5 points) Find the Jacobian of the transformation $T(r, \theta) = (2r\cos(\theta), r\sin(\theta))$.

b) (12 points) Using the transformation $S(r, \theta, z) = (2r\cos(\theta), r\sin(\theta), z)$, determine the mass of the object if its density is given by $\rho(x, y, z) = z$. *Note:* The Jacobian of S is equal to the Jacobian for the transformation T from part a).

c) (15 points) Calculate the z-coordinate of the center of mass of the object.

11) Show that

$$\lim_{(x,y)\to(5,7)} \frac{(xy - 7x + 35 - 5y)^2}{2(x-5)^3 + (y-7)^6}$$

does not exist.

12) Find the volume of the tetrahedron in the first octant spanned by the points $(0,0,0)$, $(0,0,4)$, $(0,4,0)$, and $(2,0,0)$.