READ ME: Merely finding the answer to a problem is not enough to receive full credit; you must show how you arrived at that answer.

1) This is a variant of problem 20 in section 1.4 If you don't have the text, it goes like this: A bar magnet is often modeled as a magnetic dipole with one end labeled the north pole N and the opposite end labeled the south pole S. The magnetic field for the magnetic dipole is symmetric with respect to rotation about the axis passing lengthwise through the center of the bar. Hence we can study the magnetic field by restricting ourselves to a plane with the bar magnet centered on the x-axis.

For a point P that is located a distance r from the origin where r is much greater than the length of the magnet, the **magnetic field lines** satisfy the differential equation

$$\frac{dy}{dx} = \frac{3xy}{2x^2 - y^2} \tag{1}$$

and the **equipotential lines** satisfy the equation

$$\frac{dy}{dx} = \frac{y^2 - 2x^2}{3xy} \tag{2}$$

a) (1 point) Show that the two families of curves are perpendicular where they intersect (this means the tangent lines are perpendicular).

b) (2 points) Use Mathematica or your favorite program to find the direction fields for both equations (1) and (2) when $-5 \leq x \leq 5$ and $-5 \leq y \leq 5$. You can provide these graphs either together or separately. What do these fields make you think of?

c) (2 points) Use Mathematica or your favorite program to find the isoclines for both equations (1) and (2) when $-5 \leq x \leq 5$ and $-5 \leq y \leq 5$. You can provide these graphs either together or separately; produce at least 5 isoclines for each equation.

2) This is a variant of problem 16 in section 1.4. Stefan's law of radiation states that the change in temperature of a body at $T(t)$ kelvins in a medium $M(t)$ kelvins is proportional to $M^4 - T^4$; that is,

$$\frac{dT}{dt} = K(M(t)^4 - T(t)^4) \tag{3}$$

where K is again a constant. Let $K = 40^{-4}$ and assume that the medium temperature is constant, $M(t) \equiv 70$ kelvins. Let $T(0) = 100$ kelvins.

a) (2 points) Use Euler's method **by hand** to estimate $T(.1)$ and $T(.2)$.

b) (2 points) Use Mathematica or your favorite program to produce a solution using Euler's method with $h = .1$, then approximate $T(1)$ and $T(2)$.

c) (3 points) Solve for T **by hand**. This uses only Calc II techniques!