Math 216 Assignment 2 Supplement

READ ME: Merely finding the answer to a problem is not enough to receive full credit; you must show how you arrived at that answer.

1) (Section 2.3, #32) There are occasions where the forcing term $Q(x)$ in a linear equation fails to be continuous because of jump discontinuities. Fortunately, we may still obtain a "reasonable" solution. For example, consider the initial value problem

$$\frac{dy}{dx} + 2y = Q(x), \quad y(0) = 0,$$

where

$$Q(x) := \begin{cases} 2, & 0 \le x \le 3, \\ -2, & x > 3. \end{cases}$$

   a) (2 points) Find the general solution for $0 \le x \le 3$.

   b) (1 point) Choose the constant in the solution of part a) so that the initial condition is satisfied.

   c) (2 points) Find the general solution for $x > 3$.

   d) (1 point) Now choose the constant in the general solution from part c) so that the solution from part b) and the solution from part c) agree at $x = 3$. By patching the two solutions together, we can obtain a continuous function that satisfies the differential equation except at $x = 3$, where its derivative is undefined.

   e) (1 point) Use Mathematica or your favorite program to sketch the graph of the solution from $x = 0$ to $x = 5$.

2) (Section 2.3 #35) Suppose a brine containing .2kg of salt per liter runs into a tank initially filled with 500L of water containing 5kg of salt. The brine enters the tank at a rate of 5L/min. The mixture, kept uniform by stirring, is flowing out at the rate of 5L/min.

a) (4 points) Find the concentration, in kilograms per liter, of salt in the tank after 10 min.

b) (5 points) After 10 min, a leak develops in the tank and an additional liter per minute of mixture flows out of the tank. What will be the concentration, in kilograms per liter, of salt in the tank 20 min after the leak develops? Use the method from **1)** above to solve.