

Math 216 Assignment 3 Supplement

READ ME: Merely finding the answer to a problem is not enough to receive full credit; you must show how you arrived at that answer.

1) Given the heat equation

$$\frac{1}{2} \frac{\partial^2 u}{\partial x^2}(x, t) = \frac{\partial u}{\partial t}(x, t)$$

with initial conditions

$$\frac{\partial u}{\partial x}(0, t) = 0, \quad \frac{\partial u}{\partial x}(\pi, t) = 0, \quad u(x, 0) = 1$$

we observed that if we assume $u(x, t) = f(x)g(t)$, then we are led to the equation

$$\frac{f''(x)}{f(x)} = \alpha = \frac{g'(t)}{2g(t)}$$

where α is a real number. We solved for g and f in the case $\alpha < 0$.

a) (6 points) Solve for u in the case where $\alpha = 0$. These are the *trivial solutions* to the heat equation.

b) (5 points) Solve for u in the case where $\alpha > 0$. You may use the notes to obtain the solution for g and the solution for f up to the point where we solved for r in the case $\alpha < 0$.

2) (#13 Section 4.10) (9 points) A mass weighing 32 lb is attached to a spring hanging from the ceiling and comes to rest at its equilibrium position. At time $t = 0$, an external force $F(t) = 3 \cos(4t)$ lb is applied to the system. If the spring constant is 5 lb/ft and the damping constant is 2-lb-sec/ft, find the steady-state solution for the system.