Name:

Math 216 Exam 1

October 8, 2015

Directions: WRITE YOUR NAME ON THIS EXAM! Except where indicated, merely finding the answer to a problem is not enough to receive full credit; you must show how you arrived at that answer.

1) (3 points each) For each differential equation, determine its order and whether it is linear or homogeneous.

a) $x^3\dfrac{d^2y}{dx^2} + 2x\dfrac{dy}{dx} + y = \cos(x)$.

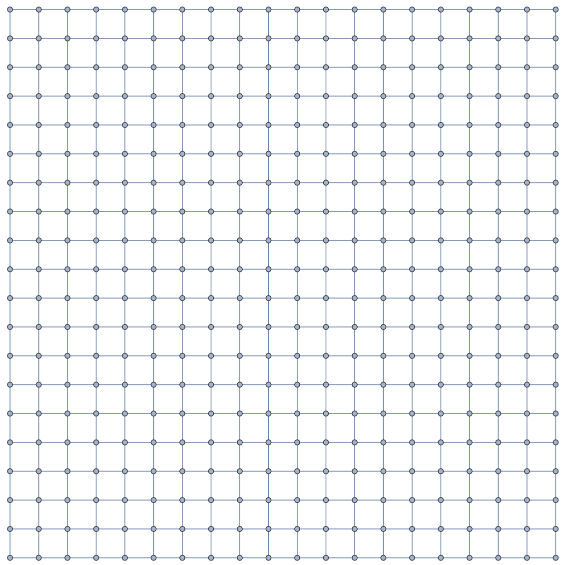
b) $y' - 2y^2x = 0$

c) $\dfrac{d^3y}{dx^2} + \ln(x+y) - 6x = 0$

2) (16 points) Given the intial value problem

$$\frac{dy}{dx} = x^3 + y^2,\ \ y(0) = 1$$

sketch an approximation of the solution on the interval $[0, 2]$ using Euler's method with step size $h = 1$. Show your work!



3) (20 points) Describe all solutions for y to the equation

$$2y + x\frac{dy}{dx} = \ln(x).$$

4) (25 points) Suppose you are given a mass-spring oscillator with no external force acting upon it. If the mass m is 1 kg, the damping coefficient is $b = 16$, and the spring constant is $k = 64$, find a formula for the position $y(t)$ of the object given that $y(0) = 4$ and $y'(0) = 3$.

5) (30 points) Find the solution to the initial value problem

$$\frac{dy}{dx} = y^2 \sin^2(x) + 7y \sin^2(x) + y^2 + 12 + 7y + 12 \sin^2(x), \; y(0) = 0.$$

on the interval $0 \leq x \leq .2$. You may assume $y(x) \geq 0$ on this interval.

BONUS: (10 points) Let I be an open interval and let f and g be two differentiable, real valued functions on I. Suppose that f and g are linearly dependent. Show that the Wronskian of f and g must be zero on I.