Name:

Math 216 Exam 2

November 5, 2015

Directions: WRITE YOUR NAME ON THIS EXAM! Except where indicated, merely finding the answer to a problem is not enough to receive full credit; you must show how you arrived at that answer.

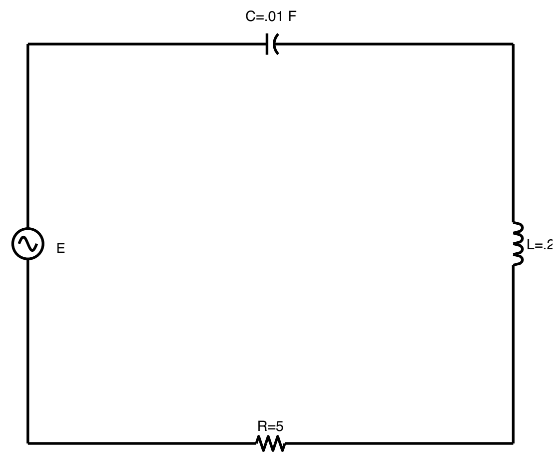
1) (3 points each) For each homogeneous second-order differential equation, determine whether it is a Cauchy-Euler equation, then write down the generic form of two linearly independent solutions to each equation.

a) $\dfrac{d^2y}{dt^2} + 6\dfrac{dy}{dt} + 9y = 0.$

b) $y'' - 2y = 0$

c) $\dfrac{1}{t}\dfrac{d^2y}{dt^2} + \dfrac{1}{t^2}\dfrac{dy}{dt} + \dfrac{y}{t^3} = 0$

2) (16 points) Suppose $E(t) = \cos(t)\sin(t)$ volts on the simple RLC circuit given below (resistance is in Ohms):



a) (4 points) Write down an equation relating all forces acting on the circuit.

b) (12 points) Determine BUT DO NOT SOLVE a differential equation whose solution would obtain the current I.

3) (36 points) Given the second order equation

$$t^2 y'' - ty' + y = t^2,$$

find all solutions for y.

4) (37 points) Suppose you are given a mass-spring oscillator with an external force of $\cos(3t)$ acting upon it. If the mass m is 1 kg, the damping coefficient is $b = 7$, and the spring constant is $k = 10$, find a formula for the position $y(t)$ of the object given that $y(0) = 4$ and $y'(0) = 3$. You may use the formula

$$\int e^{kt} \cos(mt)\ dt = \frac{e^{kt}(k\cos(mt) + m\sin(mt))}{m^2 + k^2}$$

for constants k and m.

BONUS: (10 points) Let f and g be two linearly independent solutions to the differential equation

$$y'' + p(t)y' + q(t)y = 0$$

for some real-valued, differentiable functions p and q. Show that $h(t) = f(t) + ig(t)$ is also a solution.