**Math 454/554 Assignment 1**

**Due Tuesday, 9/2**

**1)** Solve the following ODE's for $y$.

a) (separable equations) $y' = 1 + x + y^2 + xy^2$.

b) (integrating factors) $y' + \dfrac{2}{x}y = \dfrac{\cos(x)}{x^2}$, $y(\pi) = 0$, $x > 0$.

c) (homogeneous equations with constant coefficients) $y'' + 4y' + 4y = 0$, $y(-1) = 2$, $y'(-1) = 1$.

d) (variation of parameters) $y'' + 4y' + 4y = \dfrac{1}{x^2 e^{2x}}$, $x > 0$.

e) (series solutions) $xy'' + y' + xy = 0$, about $x_0 = 1$.

**Due Thursday, 9/23**

**2)** Given $f(x) = C_1 e^{irx} + C_2 e^{-irx}$, where $C_1$ and $C_2$ are arbitrary real constants and $r > 0$, show that there exist complex constants $D_1$ and $D_2$ with $f(x) = D_1 \cos(rx) + D_2 \sin(rx)$.

**3)** # 8, Section 5 (only verify when $n \neq m$).

**4)** For $0 \leq x \leq 1$, define $f_n(x) = n^2 x(1 - x^2)^n$ where $n$ is a natural number. Show that $f_n(x)$ converges to zero pointwise, but that

$$\lim_{n\to\infty} \int_0^1 f_n(x)\, dx \neq 0.$$

Conclude that $\{f_n\}_{n=1}^{\infty}$ cannot converge to 0 uniformly on $[0, 1]$.

**5)** # 5, Section 53 (you may simply assume that $f$ and $g$ are continuous on $[a, b]$).

**6)** If $-1 \leq x \leq 1$ and $n$ is a natural number, define

$$f_n(x) = \begin{cases} 0 & \text{if } \frac{1}{n} < |x|, \\ 1 - |nx| & \text{if } |x| \leq \frac{1}{n}. \end{cases}$$

Show that $f_n \to 0$ in mean ($L^2$) but that the pointwise limit is not a continuous function.

**7)** (only mandatory for graduate students) Make 3-D plots of the family of functions $\{e^{-\lambda^2 kt}\cos(\lambda x)\}_{\lambda\in\mathbb{R}}$ for at least 3 of your favorite (distinct) values of $\lambda$ using Matlab or Mathematica. For example, in Matlab, write a function mfile, say `u.m`, as follows:

```
function y = u(x,t,k)
y = exp(-λ^2*k*t)*cos(λ*x);
```

Then use the commands `meshgrid` and `surf` to make a 3-D plot.