Math 454/554 Assignment 4

Due Tuesday, 11/2

1) (# 6, Section 39)

2) (# 2, Section 40)

3) (# 3, Section 40)

4) (# 3, Section 41)

Due Thursday, 11/4

5) For $0 \leq x \leq \pi$, Let $\phi_0(x) = 1$ and $\phi_n(x) = \sin((2n-1)x)$. Show that there are constants $\{A_n\}_{n=0}^{\infty}$ and $\{B_n\}_{n=0}^{\infty}$ with

$$\sum_{n=0}^{\infty} A_n \phi_n(x) = \sum_{n=0}^{\infty} B_n \phi_n(x)$$

but that there is no value of n for which $B_n = A_n$.

6) (# 4 Section 43)

7) (only mandatory for graduate students) The nonlinear Klein-Gordon equation

$$\frac{\partial^2 u}{\partial t^2} + u^3 = \frac{\partial^2 u}{\partial x^2}$$

is a simple model for nonlinear wave interaction. Suppose $0 \leq x \leq 10$ and $t \geq 0$ and that the boundary conditions are

$$u(0,t) = u(10,t) = 0, \;\; u(x,0) = \sin(2\pi x/10), \frac{\partial u}{\partial t}(x,0) = 0.$$

a) Plot the Fourier series solution to the wave equation

$$\frac{\partial^2 u}{\partial t^2} = \frac{\partial^2 u}{\partial x^2}$$

with the boundary value data above.

b) Repeat part a) for the Klein-Gordon equation, using numerical solutions if you like. How do the graphs of your solutions differ, i.e., what is the effect of the nonlinear term? If you don't know how to solve for solutions of this equation, let me know.

8) (EXTRA CREDIT) For vectors $\vec{v} = (v_1, v_2, v_3)$ and $\vec{w} = (w_1, w_2, w_3)$ in $\mathbb{R}^3$, define the *Minkowski inner product* $\langle \vec{v}, \vec{w} \rangle_m$ of $\vec{v}$ and $\vec{w}$ to be

$$\langle \vec{v}, \vec{w} \rangle_m = -v_1 w_1 + v_2 w_2 + v_3 w_3.$$

Note that this is not actually an inner product.

Either give an example that demonstrates the following assertion or prove that the assertion is false: there exist vectors $\vec{v}, \vec{w}$, and $\vec{u}$ in $\mathbb{R}^3$ with

$$\langle \vec{v}, \vec{v} \rangle_m = \langle \vec{w}, \vec{w} \rangle_m = 0, \ \langle \vec{u}, \vec{u} \rangle_m = -1$$

$$\langle \vec{v}, \vec{w} \rangle_m = 1, \ \langle \vec{v}, \vec{u} \rangle_m = \langle \vec{w}, \vec{u} \rangle_m = 0.$$