Math 454/554 Assignment 7

Due Thursday, 12/2

1) #2, Section 72

2) Show that if $u(x) = \sqrt{x}J_0(x)$, then u satisfies the differential equation

$$x^2 u'' + \left(x^2 + \frac{1}{4}\right) u = 0.$$

3) #8, Section 75

4) (only mandatory for graduate students)

a) Use Mathematica or Matlab or whatever you prefer to plot the Bessel function $J_0(x)$ on the interval $[0, 20]$. In Mathematica, you can use the command "BesselJ." Make a rough estimate of the first three zeros of $J_0(x)$.

b) Actually determine the first three zeros. In Mathematica, you can use the command "BesselJZero."

c) If the zeros are z_1, z_2, and z_3, plot $J_0(z_1x)$, $J_0(z_2x)$, and $J_0(z_3x)$ on the interval $[0, 1]$.

Due Tuesday, 12/7

5) #4, Section 85

6) #2, Section 84 (see the first example in this section)

7) #4, Section 79

8) (only mandatory for graduate students) Recall that for $f, g, p \in C([a, b])$, we denote by $\langle f, g\rangle_p$ the bracket (or inner product)

$$\langle f, g\rangle_p = \int_a^b f(x)g(x)p(x)\ dx.$$

Recall from problem # 4 above that $\{z_i\}_{i=1}^3$ are the first three positive zeros of J_0.

a) Using Mathematica or Matlab or whatever you prefer, calculate the brackets

$$\langle J_0(z_i x), J_0(z_j x)\rangle_x$$

on the interval $[0, 1]$ for $1 \leq i, j \leq 3$ and $i \neq j$.

b) Find constants c_1, c_2, and c_3 so that

$$\left\langle \frac{J_0(z_i x)}{c_i} \frac{J_0(z_i x)}{c_i} \right\rangle_x = 1$$

on the interval $[0, 1]$.