

Math 454/554 Assignment 8

Due Tuesday, 12/14

1) Verify that if $n \neq m$ are integers, then

$$\int_{-\pi}^{\pi} e^{inx} e^{-imx} dx = 0.$$

2) If F_4 is the discrete 4×4 Fourier transform, check that $F_4^* F_4 = 4I_4$ where I_4 is the 4×4 identity matrix.

3) If F_8 is the discrete 8×8 Fourier transform, use the Fast Fourier Transform to compute $y = F_8 c$ where

$$c = \begin{pmatrix} 0 \\ 1 \\ 0 \\ 1 \\ 0 \\ 1 \\ 0 \\ 1 \end{pmatrix}.$$

Suggestion: Use Mathematica to check your answer. The command is “Fourier.”

4) For any vector $c \in \mathbb{C}^n$, let \bar{c} be the vector whose components are the complex conjugates of the entries of c . If “ \cdot ” is the usual dot product, prove that

$$c \cdot \bar{c} = \frac{1}{n} (F_n c) \cdot \overline{F_n c}.$$

where F_n is the discrete $n \times n$ Fourier transform. *Hint:* for any $n \times n$ matrix A with complex coefficients,

$$Ax \cdot \bar{y} = x \cdot \overline{A^* y}$$

for $x, y \in \mathbb{C}^n$.

5) Compute the Fourier transform of the function $f(x) = e^{-\pi x^2}$.

6) (only mandatory for graduate students- courtesy of Joan Remski) Let

$$y = \begin{pmatrix} 5 \\ \sqrt{2} + 8 \cos\left(\frac{\pi}{8}\right) \\ -2 + 4\sqrt{2} \\ \sqrt{2} + 2 \sin\left(\frac{\pi}{8}\right) \\ -3 \\ -\sqrt{2} - 8 \sin\left(\frac{\pi}{8}\right) \\ 2 - \sqrt{2} \\ -\sqrt{2} - 2 \cos\left(\frac{\pi}{8}\right) \end{pmatrix}.$$

Recall that the absolute value of a complex number $z = a + ib$ is $|z| = \sqrt{a^2 + b^2}$.

a) Use Matlab or Mathematica or whatever program you prefer to solve for c in the equation $y = F_8 c$. The relevant Mathematica command is “Inverse Fourier.”

b) Find the three smallest (in absolute value) coefficients of your answer from part a) and replace them with zero in the new vector

$$\tilde{c} = \begin{pmatrix} c_0 \\ c_1 \\ c_2 \\ c_3 \\ c_4 \\ c_5 \\ c_6 \\ c_7 \end{pmatrix}.$$

Then use the Fourier transform F_8 to find $\tilde{y} = F_8 \tilde{c}$. Compare this with your original vector.

c) Plot y and \tilde{y} on the same axes (use the real part of a component as an x -coordinate and the imaginary part as the y -coordinate).