Math 331 Assignment 1

Due Thursday, September 20

In this homework set, you are free to use all the primitives, axioms, definitions, and theorems proved in Chapter 1.

1) Look at the definitions in Chapter 1, Exercises 3 and 8; internalize them for later use. Then answer #11 in the Chapter 1 Exercises: Do you think that the Euclidean parallel postulate is "obvious"? Write a brief essay (i.e., a couple of sentences) explaining your answer.

2) (#7, Chapter 1 Exercises) If S and T are any sets, their *union* $(S \cup T)$ and *intersection* $(S \cap T)$ are defined as follows:

(i) Something belongs to $S \cup T$ if and only if it belongs either to S or to T (or to both of them).

(ii) Something belongs to $S \cap T$ if and only if it belongs both to S and to T.

Given two points A and B, consider the two rays $\overrightarrow{AB}$ and $\overrightarrow{BA}$. Draw diagrams to show that $\overrightarrow{AB} \cup \overrightarrow{BA} = \overleftrightarrow{AB}$ and $\overrightarrow{AB} \cap \overrightarrow{BA} = AB$. What additional axioms about the undefined term "between" must we assume in order to be able to *prove* these equalities?

3) (#14, Chapter 1 Exercises) In this exercise, we will review several basic Euclidean constructions with a straightedge and compass. Such constructions fascinated mathematicians from ancient Greece until the nineteenth century, when all classical construction problems were finally solved.

a) Given a segment AB. Construct the perpendicular bisector of AB. (Hint: Make AB a diagonal of a rhombus, as in Figure 1.23.)

b) Given a line l and a point P lying on l. Construct the line through P perpendicular to l. (Hint: Make P the midpoint of a segment of l.)

c) Given a line l and a point P *not* lying on l. Construct the line through P perpendicular to l. (Hint: Construct isosceles triangle $\triangle ABP$ with base AB on l and use a).)

d) Given a line l and a point P not lying on l. Construct a line through P parallel to l. (Hint: Use b) and c).)

e) Construct the bisecting ray of an angle (Hint: Use the Euclidean theorem that the perpendicular bisector of the base on an isosceles triangle is also the angle bisector of the angle opposite the base.)

4) (#16, Chapter 1 Exercises) The straightedge you used in the previous exercises was supposed to be *unruled* (if it did have marks on it, you weren't supposed to use them). Now, however, let us mark two points on the straightedge so as to mark off a certain distance d. Archimedes showed how we can then trisect an arbitrary angle.

For any angle, draw a circle γ of radius d centered at the vertex O of the angle. This circle cuts the sides of the angle at points A and B. Place the marked straightedge so that one mark gives a point C on line $\overleftrightarrow{OA}$ such that O is between C and A, the other mark gives a point D on circle γ, and the straightedge must simultaneously rest on the point B, so that B, C, and D are collinear (Figure 1.25). Prove that $\sphericalangle COD$ so constructed is one-third of $\sphericalangle AOB$. (Hint: Use Euclidean theorems on exterior angles and isosceles triangles.) *Note:* one of the crowning achievements of Abstract Algebra is the proof that one cannot trisect an arbitrary angle using only straightedge and compass!