Math 331 Assignment 2

Due Thursday, October 4

In this homework set, you are free to use all the primitives, axioms, definitions, and theorems proved in Chapter 2, and the betweenness axioms and theorems of Chapter 3.

1) (#6, Chapter 2 Exercises) Give formal proofs of Propositions 2.3, 2.4, and 2.5.

2) (#9, Chapter 2 Exercises) In each of the following interpretations of the undefined terms, which of the axioms of incidence geometry are satisfied and which are not? (You don't have to prove it, just list which axioms are satisfied and which are not.) Tell whether each interpretation has the elliptic, Euclidean, or hyperbolic parallel property.

(c) Fix a circle in the Euclidean plane. Interpret "point" to mean a Euclidean point inside the circle, interpret "line" to mean a chord of the circle, and let "incidence" mean that the the point lies on the chord. (A *chord* of a circle is a segment whose endpoints lie on the circle.)

(d) Fix a sphere in Euclidean 3-space. Two points on the sphere are called *antipodal* if they lie on a diameter of the sphere; e.g., the north and south poles are antipodal. Interpret a "point" to be a set $\{P, P'\}$ consisting of two points on the sphere that are antipodal. Interpret a "line" to be a great circle on the sphere. Interpret a "point" $\{P, P'\}$ to "lie on" a "line" C if both P and P' lie on C (actually, if one lies on C, then so does the other, by the definition of "great circle.")

3) (#11, Chapter 2 Exercises) Invent a model of incidence geometry that has neither the elliptic, hyperbolic, nor Euclidean parallel properties. These properties refer to any line l and any point P not on l. Invent a model that has different parallelism properties for different choices of l and P. (Hint: Five points suffice for a finite example, or you could find a suitable piece of the Euclidean plane for an infinite example, or you could refer to a previous exercise. Or invent a fourth example.)

4) At the bottom of page 87, Greenberg blithely declares, 'Hopefully, with this model, you see that there is nothing mysterious about the "line at infinity"...nor is there any mystery about the "point at infinity" common to all the parallel affine lines $ax + by = t$...' Do you find that there is nothing mysterious about these concepts? Why or why not? Write a couple of sentences.

5) (#15, Chapter 2 Exercises) a) Four distinct points, no three of which are collinear, are said to form a *quadrangle*. Let $\mathcal{P}$ be a model of incidence geometry for which every line has at least three distinct points lying on it. Show that a quadrangle exists in $\mathcal{P}$.

6) (#5, Chapter 3 Exercises on Betweenness) Given $A * B * C$. Prove that $\overrightarrow{AB} = \overrightarrow{AC}$, completing the proof of Proposition 3.6. Deduce that every ray has a *unique* opposite ray.

7) (#9, Chapter 3 Exercises on Betweenness) Given a line l , a point A on l, and a point B not on l. Then every point of the ray $\overrightarrow{AB}$ (except A) is on the same side of l as B. (Hint: Use an RAA argument.)