Math 331 Assignment 3

Due Tuesday, November 6

In this homework set, you are free to use all the primitives, axioms, definitions, and results proved in Chapters 2-4.

1) (#2, Chapter 4 Exercises) The following purports to be a proof in neutral geometry of the SAA congruence criterion. Find the step in the proof that is not valid in neutral geometry and indicate for which special Hilbert planes the proof is valid (see figure 4.5, p. 167).

Given $AC \cong DF$, $\sphericalangle A \cong \sphericalangle D$, $\sphericalangle B \cong \sphericalangle E$. Then $\sphericalangle C \cong \sphericalangle F$ since

$$\begin{aligned}(\sphericalangle C)^\circ &= 180^\circ - (\sphericalangle A)^\circ - (\sphericalangle B)^\circ \\ &= 180^\circ - (\sphericalangle D)^\circ - (\sphericalangle E)^\circ = (\sphericalangle F)^\circ\end{aligned}$$

2) (#4, Chapter 4 Exercises) Prove Proposition 4.2. (Hint: See Figure 4.6. On the ray opposite to $\overrightarrow{AC}$, lay off segment AD congruent to $A'C'$. First prove $\triangle DAB \cong \triangle C'A'B'$; then use isosceles triangles and a congruence criterion to conclude.)

3) (#7, Chapter 4 Exercises) Prove that every acute angle has a complementary angle and that if complements of two acute angles are congruent, then the acute angles are congruent.

4) (#10, Chapter 4 Exercises) Prove Proposition 4.7. Deduce as a corollary that transitivity of parallelism is equivalent to Hilbert's Euclidean parallel postulate.

5) (#12, Chapter 4 Exercises) Prove Proposition 4.9. (Hint: Be sure to prove both implications here. It may be helpful to use Propositions 4.7 and 4.8.)

6) (#14, Chapter 4 Exercises) The ancient Greek mathematician Heron gave an elegant proof of the triangle inequality different from the one in the text. In order to prove $\overline{AB} + \overline{AC} > \overline{BC}$, he bisected $\sphericalangle A$. He let the bisector meet BC at point D, which we justify via the crossbar theorem. He then applied the exterior angle theorem and Proposition 4.5 to triangles $\triangle BAD$

and $\triangle CAD$. Fill in the details of this argument. (Hint: draw a diagram to help you.)

7) (#18, Chapter 4 Exercises) In any Hilbert plane, prove that *every triangle has an inscribed circle*- more specifically, prove that *the three angle bisectors are concurrent* in a point P (called the *incenter*) interior to the triangle which is equidistant from the sides of the triangle-i.e., the perpendiculars dropped from P to the sides are congruent- so that the circle with center P and radius equal to any of those perpendiculars is tangent to the sides of the triangle. (Hint: Show first that two angle bisectors must meet at a point P interior to the triangle; then show by congruent triangles that P is equidistant from the sides and lies on the third angle bisector.)

8) (#21, Chapter 4 Exercises) The sphere, with "lines" interpreted as great circles, is not a model of neutral geometry. Here is a proposed construction of a rectangle on a sphere. Let α, β be two circles of longitude and let them intersect the equator at A and D. Let γ be a circle of latitude in the northern hemisphere intersecting α and β at two other points, B and C. Since circles of latitude are perpendicular to circles of longitude, the quadrilateral with vertices $ABCD$ and sides the arcs of α, γ, and β and the equator traversed in going from A north to B east to C south to D west to A should be a rectangle. Explain why this construction doesn't work.