Name:

Math 331 Exam 2

December 4, 2012

1) (10 points) True or False: 2 points each. No justification necessary.

a) Every Lambert quadrilateral in a Hilbert plane is a Saccheri quadrilateral.

b) For any triangle, the greater side lies opposite the greater angle.

c) In hyperbolic geometry, any two similar triangles are congruent.

d) In neutral geometry, if two lines are parallel, then the lines have congruent alternate interior angles.

e) If a point P lies inside a circle C in the Euclidean plane, then the inverse of P with respect to C lies outside C.

2) a) (5 points) State the alternate interior angle theorem.

b) (8 points) If two lines ℓ and m are perpendicular to a given third line n, draw a picture and use the alternate interior angle theorem to show that ℓ and m must be parallel.

c) (5 points) State the exterior angle theorem.

3) (9 points) Give *three* statements in neutral geometry that are equivalent to Hilbert's Parallel Postulate: given any line ℓ and any point P not on line ℓ, there exists a unique line m through P parallel to ℓ.

4) a) (3 points) Define a Saccheri quadrilateral.

b) (3 points) Define a Lambert quadrilateral.

c) (6 points) State the acute angle hypothesis for a Hilbert plane. What does this hypothesis imply for Saccheri quadrilaterals?

5) Let $\mathbb{D}$ denote the unit disk in the Euclidean plane.

a) (6 points) Define the two types of hyperbolic lines in $\mathbb{D}$, then draw an example of each type.

b) (4 points) Given a line ℓ and a point P not on ℓ in the disk model of the hyperbolic plane, draw at least two distinct parallel lines to ℓ through P.

c) (8 points) Given a line ℓ and a point P not on ℓ in the disk model of the hyperbolic plane, define a *limiting parallel ray* of ℓ through P, then draw a limiting parallel ray to ℓ through P.

6) (18 points) Given a circle C in a Euclidean Hilbert plane and a point O not on the circle, prove that if two lines through O intersect C in pairs of points (P_1, P_2) and (Q_1, Q_2), respectively, then $(\overline{OP_1}) \cdot (\overline{OP_2}) = (\overline{OQ_1}) \cdot (\overline{OQ_2})$.

7) (25 points) Assume $\triangle ABC$ is a right triangle in a Euclidean Hilbert plane. Prove the Pythagorean theorem for $\triangle ABC$.