Math 331 Worksheet 7

Note: None of these problems are for a grade, though you may obtain extra credit for writing up proofs on the board.

1) a) Greenberg notes that in elliptic geometry, the exterior angle theorem (Theorem 4.2) does not hold due to the absence of betweenness axiom 4. Go back to the betweenness axioms and decide which, if any, of the axioms hold for the real projective plane.

b) In yet another wonderful aside, Greenberg backhandedly comments, "Many reputable writers mistakenly state that to fill this gap (proof of Corollary 1) in Euclid one must add an axiom that 'lines are infinite in extent'- whatever that may mean. All that is needed are the betweenness axioms and their consequences." Look over the proof of Corollary 1 and note that the only betweenness axiom that is NOT used is the one whose consequence is that every line has infinitely many points! What do you think Greenberg is talking about?

2) Complete the proof of SAA (Proposition 4.1) by justifying every step given in Exercise 3 in the text.

3) Prove the following propositions from the book. You may use any and all betweenness, congruence, and incidence axioms, plus prior consequences deduced in the text.

a) (Proposition 4.4) Every angle has a unique bisector.

b) (Proposition 4.4) Every segment has a unique perpendicular bisector.

4) Joe Biden has suggested the following alternate proof of the triangle inequality to me. Either find the flaw in Joe's reasoning or justify every step: Consider a triangle $\triangle ABC$. We want to show that $\overline{AC} < \overline{AB} + \overline{BC}$.

- There is a unique perpendicular line to AC that contains the point B. Call this line l and let P be the point of intersection of l with AC.
- $\sphericalangle BPC > \sphericalangle ABP$.
- Therefore, $\sphericalangle BPA > \sphericalangle ABP$.

- Then $AB > AP$.
- So $\overline{AB} > \overline{AP}$.
- Similarly, $\overline{BC} > \overline{CP}$.
- Now $\overline{AC} = \overline{AP} + \overline{PC} < \overline{AB} + \overline{BC}$.