Math 331 Worksheet 9

Note: None of these problems are for a grade, though you may obtain extra credit for writing up proofs on the board.

1) These are results in Euclidean geometry. As such, you are allowed to assume the Euclidean parallel postulate (and all its equivalences) holds.

a) Prove the Pythagorean theorem. You may use the Parallel Projection Theorem proved in HW# 4. See p. 233, question 13.

b) Prove the angle addition formulas $\sin(A+B) = \sin(A)\cos(B) + \sin(B)\cos(A)$ and $\cos(A+B) = \cos(A)\cos(B) - \sin(A)\sin(B)$.

2) These are results in Hyperbolic geometry. As such, you may assume Hilbert's Hyperbolic Axiom of Parallels is in effect.

a) Prove the hyperbolic law of cosines and the hyperbolic law of sines: If $\triangle ABC$ is a triangle in $\mathbb{D}$ and a, b, and c are the opposite sides to A, B, and C, respectively, then

$$\cosh(c) = \cosh(a)\cosh(b) - \sinh(a)\sinh(b)\cos(C)$$

and

$$\frac{\sin(A)}{\sinh(a)} = \frac{\sin(B)}{\sinh(b)} = \frac{\sin(C)}{\sinh(c)}.$$

Recall that $\cosh(x) = \dfrac{e^x + e^{-x}}{2}$ and $\sinh(x) = \dfrac{e^x - e^{-x}}{2}$. You may use theorem 10.3 in the text (p. 492) and any identities listed on page 490.

b) Let a, b, c, d be real numbers and suppose $ad - bc \neq 0$. Show that if we regard a point (x, y) in $\mathbb{D}$ as a complex number $x + iy$, then the map

$$f(z) = \frac{az + b}{cz + d}$$

preserves the hyperbolic distances between two points (x, y) and (s, t) in $\mathbb{D}$.