Math 227 Assignment 1

Due Thursday, January 15

1) (3 points) For ONE TIME ONLY: solve the system of linear equations

$$\begin{aligned} 22x - 3y &= -15 \\ -4x + 17y &= 32 \end{aligned}$$

BY HAND for x and y. You may convert the system to a matrix first, but the calculations should be yours, not a computer's.

2) Solve the system of linear equations

$$\begin{aligned} 2x - 3y + z &= 10 \\ -9x + 3y + 4z &= 2 \\ 6x - 7y - 4z &= 8 \end{aligned}$$

by

a) (2 points) writing down an augmented matrix that corresponds to the equations and then

b) (2 points) solving the system by putting the matrix in row-reduced echelon form and then reading off the solutions.

3) From exercise 33 in section 1.2, an **interpolating polynomial** for a set of points is a polynomial whose graph passes through every point in the set.

a) (2 points) Find a quadratic interpolating polynomial $ax^2 + bx + c$ for the points $(1, 2), (2, 3)$, and $(3, 9)$; that is, find a, b, and c with

$$\begin{aligned} c + b \cdot 1 + a \cdot (1)^2 &= 2 \\ c + b \cdot 2 + a \cdot (2)^2 &= 3 \\ c + b \cdot 3 + a \cdot (3)^2 &= 9 \end{aligned}$$

by finding an augmented matrix that describes the system of linear equations, then putting the matrix in row-reduced echelon form.

b) (3 points) Show that through any 3 points $(1, y_1)$, $(2, y_2)$, and $(3, y_3)$, there is an interpolating polynomial of degree no more than two. *Hint:* Wolfram Alpha can do this problem.

c) (4 points) When is there an interpolating polynomial through the points (x_1, y_1), (x_2, y_2), and (x_3, y_3)? When is there a *quadratic* interpolating polynomial through the points (x_1, y_1), (x_2, y_2), and (x_3, y_3)? *Hint:* Wolfram Alpha can also do this problem, but you'll have to interpret the answer!

4) (3 points) Is it possible for a system of linear equations with more equations than unknowns to have a unique solution? What about fewer equations than unknowns? If your answer to either question is "yes," provide an example.

5) a) (3 points) Let

$$v_1 = \begin{bmatrix} 0 \\ 1 \\ 4 \end{bmatrix}, v_2 = \begin{bmatrix} -4 \\ -5 \\ 7 \end{bmatrix}, v_3 = \begin{bmatrix} 14 \\ 10 \\ 8 \end{bmatrix}, b = \begin{bmatrix} 16 \\ 18 \\ 19 \end{bmatrix}.$$

Show that b is in $\text{span}\{v_1, v_2, v_3\}$ by

i) writing down a system of linear equations that describes when b is in $\text{span}\{v_1, v_2, v_3\}$,

ii) producing an augmented matrix for the system in a), then

iii) solving the system by putting the matrix in row-reduced echelon form and reading off the solutions.

b) (3 points) Let v_1, v_2, and v_3 be three nonzero vectors in $\mathbb{R}^3$. Suppose v_2 is not a scalar multiple of either v_1 or v_3 and v_3 is not a scalar multiple of either v_1 or v_2. Does it follow that every vector in $\mathbb{R}^3$ is in $\text{span}\{v_1, v_2, v_3\}$?