

## Math 227 Assignment 2

Due Thursday, January 22

1) More on span:

a) (2 points) Let

$$v_1 = \begin{bmatrix} 4 \\ -8 \\ 9 \\ 6 \end{bmatrix}, v_2 = \begin{bmatrix} 0 \\ 5 \\ 11 \\ -6 \end{bmatrix}, \text{ and } v_3 = \begin{bmatrix} 12 \\ 2 \\ -4 \\ -7 \end{bmatrix}.$$

Show that  $v = \begin{bmatrix} 128 \\ -11 \\ -55 \\ -40 \end{bmatrix}$  is in  $\text{span}\{v_1, v_2, v_3\}$  by writing down an appropriate matrix and row-reducing it.

b) (3 points) Let

$$v_1 = \begin{bmatrix} 1 \\ 2 \\ 3 \\ 4 \end{bmatrix} \text{ and } v_2 = \begin{bmatrix} -4 \\ 0 \\ 6 \\ 7 \end{bmatrix}.$$

Find a vector  $v$  in  $\text{span}\{v_1, v_2\}$  that is neither a scalar multiple of  $v_1$  nor a scalar multiple of  $v_2$ . Then find a vector  $v$  that is NOT in  $\text{span}\{v_1, v_2\}$  by writing down an appropriate matrix and row reducing it.

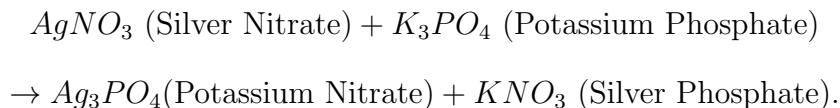
c) (4 points) Let  $v_1, v_2, v_3$ , and  $v_4$  be vectors in  $\mathbb{R}^4$ . Suppose  $v_1$  is not a scalar multiple of  $v_2$ ,  $v_3$  is not a scalar multiple of  $v_4$ , and that  $\text{span}\{v_1, v_2\} \neq \text{span}\{v_3, v_4\}$ . Show, via a choice of vectors for  $v_1, v_2, v_3$ , and  $v_4$ , that  $\text{span}\{v_1, v_2, v_3, v_4\}$  need not equal  $\mathbb{R}^4$  by writing down an appropriate matrix and row-reducing it.

2) Balance each chemical equation by i) writing down an augmented matrix through which the solution can be obtained and then ii) row-reducing the matrix and finding the solution.

a) (3 points)



b) (3 points)



3) Consider the system of linear equations

$$3x - 8y + z = 4 \\ -20x - 5y - 6z = 2.$$

Find all solutions to the system by

a) (2 points) Finding a single solution by setting one of the variables  $x$ ,  $y$ , or  $z$  equal to zero, forming an augmented matrix for the resulting system, and then row-reducing the matrix for a unique solution in  $x$ ,  $y$ , and  $z$ ;

b) (2 points) solving the homogeneous system

$$3x - 8y + z = 0 \\ -20x - 5y - 6z = 0.$$

by forming the augmented matrix and then row-reducing the matrix for solutions of the homogeneous system;

c) (1 point) solving the system by adding your solutions from parts a) and b) together;

d) (2 points) checking your work by forming the augmented matrix of the original system, then row-reducing. Which method do you like better?

4) (3 points) Let  $A$  be an  $m \times n$  matrix,  $x$  a vector in  $\mathbb{R}^n$ , and  $b$  a vector in  $\mathbb{R}^m$ . Show that if  $x_1$  in  $\mathbb{R}^n$  is a solution to  $Ax = b$  and  $x_2$  is a solution to  $Ax = \vec{0}$ , then  $x_1 + x_2$  is a solution to  $Ax = b$ .