Math 227 Assignment 2

Due Thursday, January 22

1) More on span:

a) (2 points) Let

$$v_1 = \begin{bmatrix} 4 \\ -8 \\ 9 \\ 6 \end{bmatrix}, \; v_2 = \begin{bmatrix} 0 \\ 5 \\ 11 \\ -6 \end{bmatrix}, \text{ and } v_3 = \begin{bmatrix} 12 \\ 2 \\ -4 \\ -7 \end{bmatrix}.$$

Show that $v = \begin{bmatrix} 128 \\ -11 \\ -55 \\ -40 \end{bmatrix}$ is in $span\{v_1, v_2, v_3\}$ by writing down an appropriate matrix and row-reducing it.

b) (3 points) Let

$$v_1 = \begin{bmatrix} 1 \\ 2 \\ 3 \\ 4 \end{bmatrix} \text{ and } v_2 = \begin{bmatrix} -4 \\ 0 \\ 6 \\ 7 \end{bmatrix}.$$

Find a vector v in $span\{v_1, v_2\}$ that is neither a scalar multiple of v_1 nor a scalar multiple of v_2. Then find a vector v that is NOT in $span\{v_1, v_2\}$ by writing down an appropriate matrix and row reducing it.

c) (4 points) Let v_1, v_2, v_3, and v_4 be vectors in $\mathbb{R}^4$. Suppose v_1 is not a scalar multiple of v_2, v_3 is not a scalar multiple of v_4, and that $span\{v_1, v_2\} \neq span\{v_3, v_4\}$. Show, via a choice of vectors for v_1, v_2, v_3, and v_4, that $span\{v_1, v_2, v_3, v_4\}$ need not equal $\mathbb{R}^4$ by writing down an appropriate matrix and row-reducing it.

2) Balance each chemical equation by i) writing down an augmented matrix through which the solution can be obtained and then ii) row-reducing the matrix and finding the solution.

a) (3 points)

C_2H_6 (Ethane) + O_2 (Oxygen) $\rightarrow$ CO_2 (Carbon Dioxide) + H_2O (Water)

b) (3 points)

$$AgNO_3 \text{ (Silver Nitrate)} + K_3PO_4 \text{ (Potassium Phosphate)}$$

$$\rightarrow Ag_3PO_4 \text{(Potassium Nitrate)} + KNO_3 \text{ (Silver Phosphate)}.$$

3) Consider the system of linear equations

$$3x - 8y + z = 4$$

$$-20x - 5y - 6z = 2.$$

Find all solutions to the system by

a) (2 points) Finding a single solution by setting one of the variables x, y, or z equal to zero, forming an augmented matrix for the resulting system, and then row-reducing the matrix for a unique solution in x, y, and z;

b) (2 points) solving the homogeneous system

$$3x - 8y + z = 0$$

$$-20x - 5y - 6z = 0.$$

by forming the augmented matrix and then row-reducing the matrix for solutions of the homogeneous system;

c) (1 point) solving the system by adding your solutions from parts a) and b) together;

d) (2 points) checking your work by forming the augmented matrix of the original system, then row-reducing. Which method do you like better?

4) (3 points) Let A be an $m \times n$ matrix, x a vector in $\mathbb{R}^n$, and b a vector in $\mathbb{R}^m$. Show that if x_1 in $\mathbb{R}^n$ is a solution to $Ax = b$ and x_2 is a solution to $Ax = \vec{0}$, then $x_1 + x_2$ is a solution to $Ax = b$.