Math 227 Assignment 3

Due Thursday, January 29

1) Determine whether the vectors

$$v_1 = \begin{bmatrix} 2 \\ 4 \\ 5 \end{bmatrix}, v_2 = \begin{bmatrix} -6 \\ 13 \\ 11 \end{bmatrix}, v_3 = \begin{bmatrix} -4 \\ 17 \\ 10 \end{bmatrix}$$

are linearly independent by

a) (1 point) writing down a matrix that corresponds to the equation $xv_1 + yv_2 + zv_3 = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$, and then

b) (1 point) row-reducing the given matrix and drawing your conclusions from the row-reduced echelon form.

2) For the following transformations, first check that the transformation is linear, then find the standard matrix of the transformation. For part c), also check that $P \circ P = P$.

a) (3 points) $T : \mathbb{R}^3 \to \mathbb{R}^2$, $T(x, y, z) = (x - 5y + 34z, 2x + 3z - 8y)$.

b) (3 points) $S : \mathbb{R} \to \mathbb{R}^4$, $S(t) = (5t, -6t, 3t/2, 0)$.

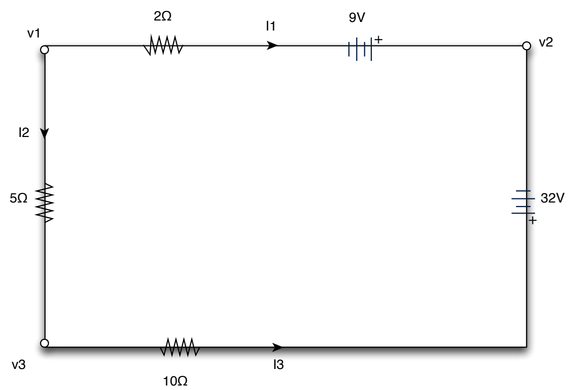
c) (4 points) $P : \mathbb{R}^2 \to \mathbb{R}^2$, $P(x, y) = ((x + y)/2, (x + y)/2)$.

3) a) (3 points) Let $\{v_1, v_2, v_3\}$ in $\mathbb{R}^n$ be linearly independent. Prove that $\{v_1 + v_2, v_2 + v_3, v_3 + v_1\}$ is linearly independent.

b) (2 points) Let $T : \mathbb{R}^n \to \mathbb{R}^m$ be linear. Prove that for all linearly dependent collections of vectors $\{v_1, v_2, \ldots, v_k\}$ in $\mathbb{R}^n$, $\{Tv_1, Tv_2, \ldots, Tv_k\}$ is also linearly dependent.

4) (4 points each) Calculate the values of the indicated currents in the diagrams below.

a)



b)